The background features several faint, light-gray geometric patterns on a dark gray background. These include concentric circles, arcs, and circular paths with arrows indicating direction. Some of these paths are marked with degree values, such as 150, 160, 170, 180, 190, 200, 210, 220, 230, 240, 250, and 260.

A CORRESPONDENCE BETWEEN PROJECTIVE GEOMETRY AND CIRCLE GEOMETRY

OPAL GRAHAM

FLASH TALK

SPRING 2019



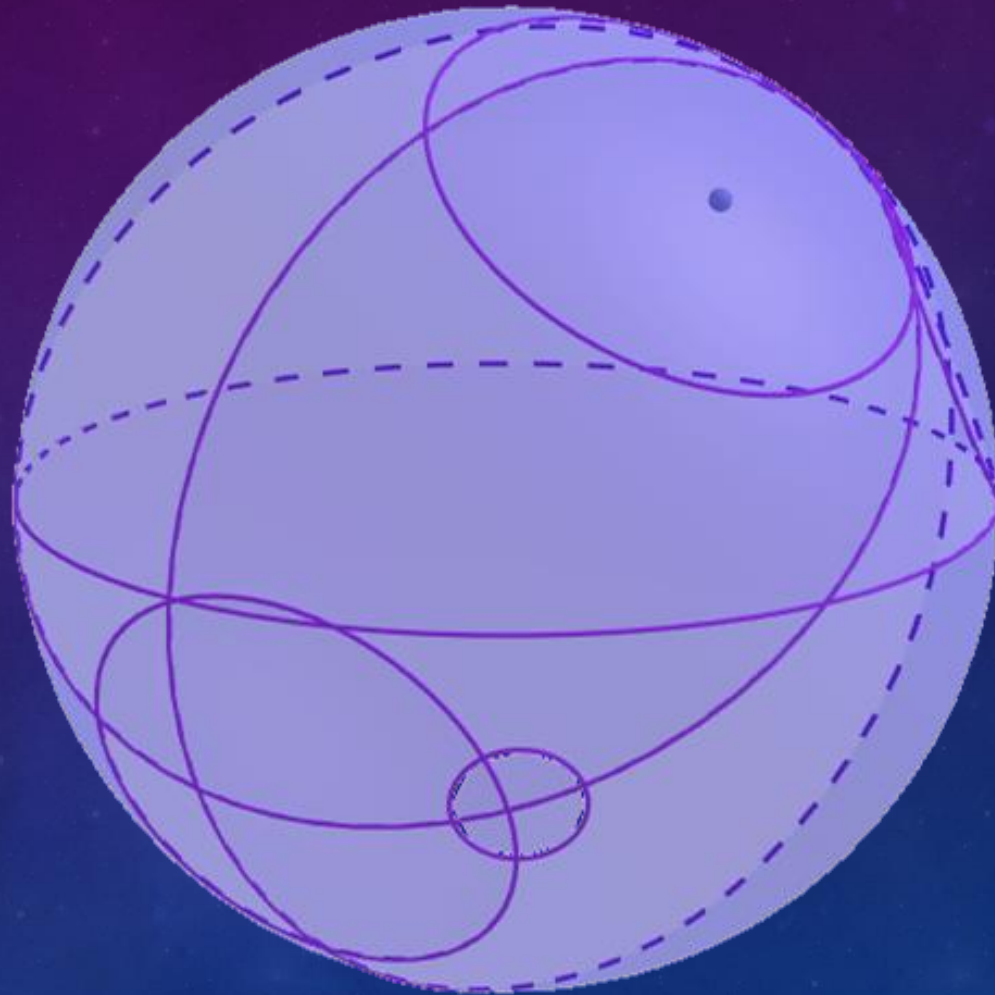
FLORIDA STATE UNIVERSITY
Mathematics

CIRCLE SPACE $\mathcal{C}(\mathbb{S}^2)$



CIRCLE SPACE $\mathcal{C}(\mathbb{S}^2)$

- Elements are circles of radius $0 \leq r \leq \pi$ in \mathbb{S}^2 .



PROJECTIVE SPACE \mathbb{RP}^3

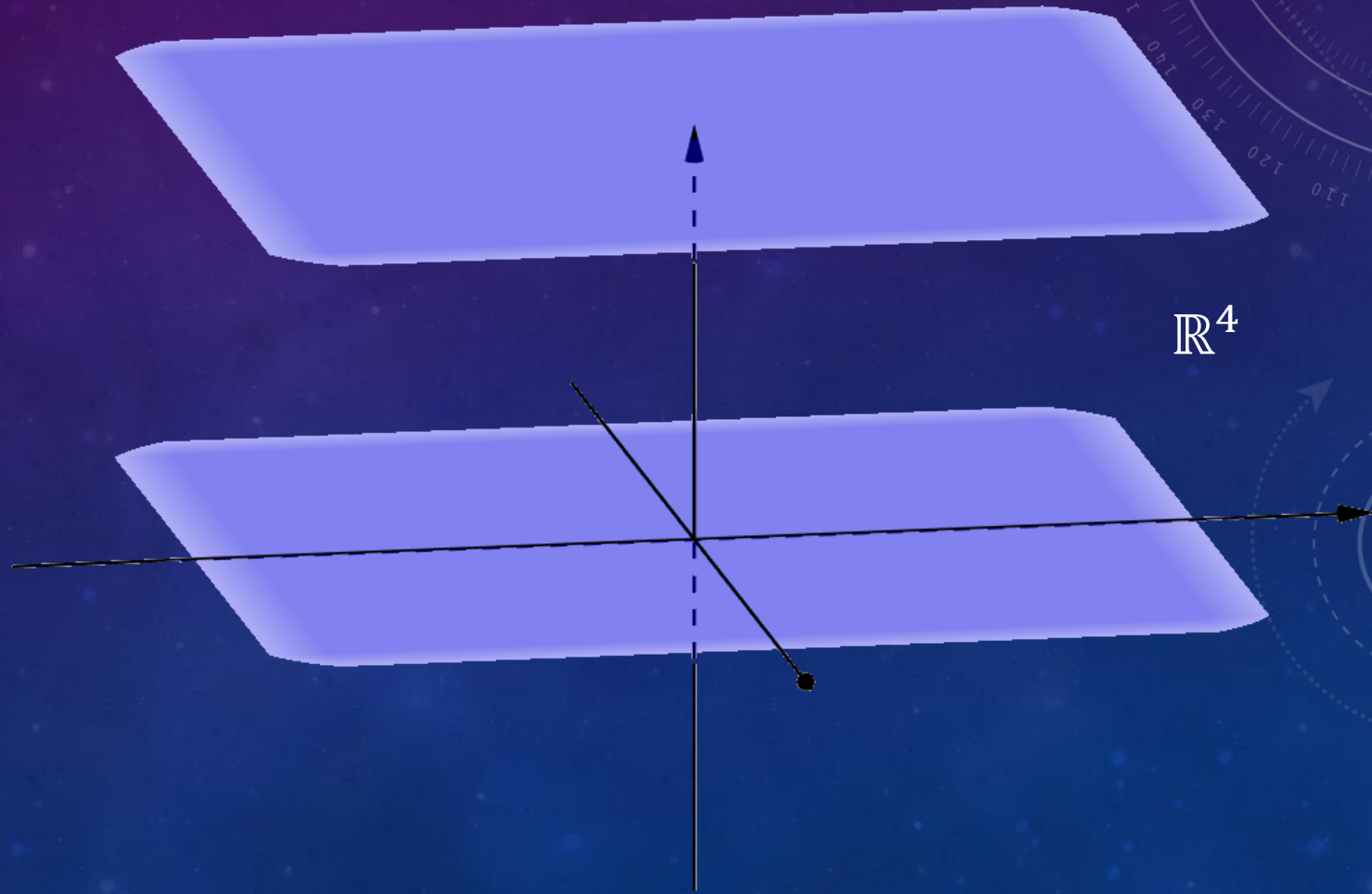


PROJECTIVE SPACE \mathbb{RP}^3

- Elements are points in $\mathbb{E}^3 \cup \mathbb{RP}^2$.

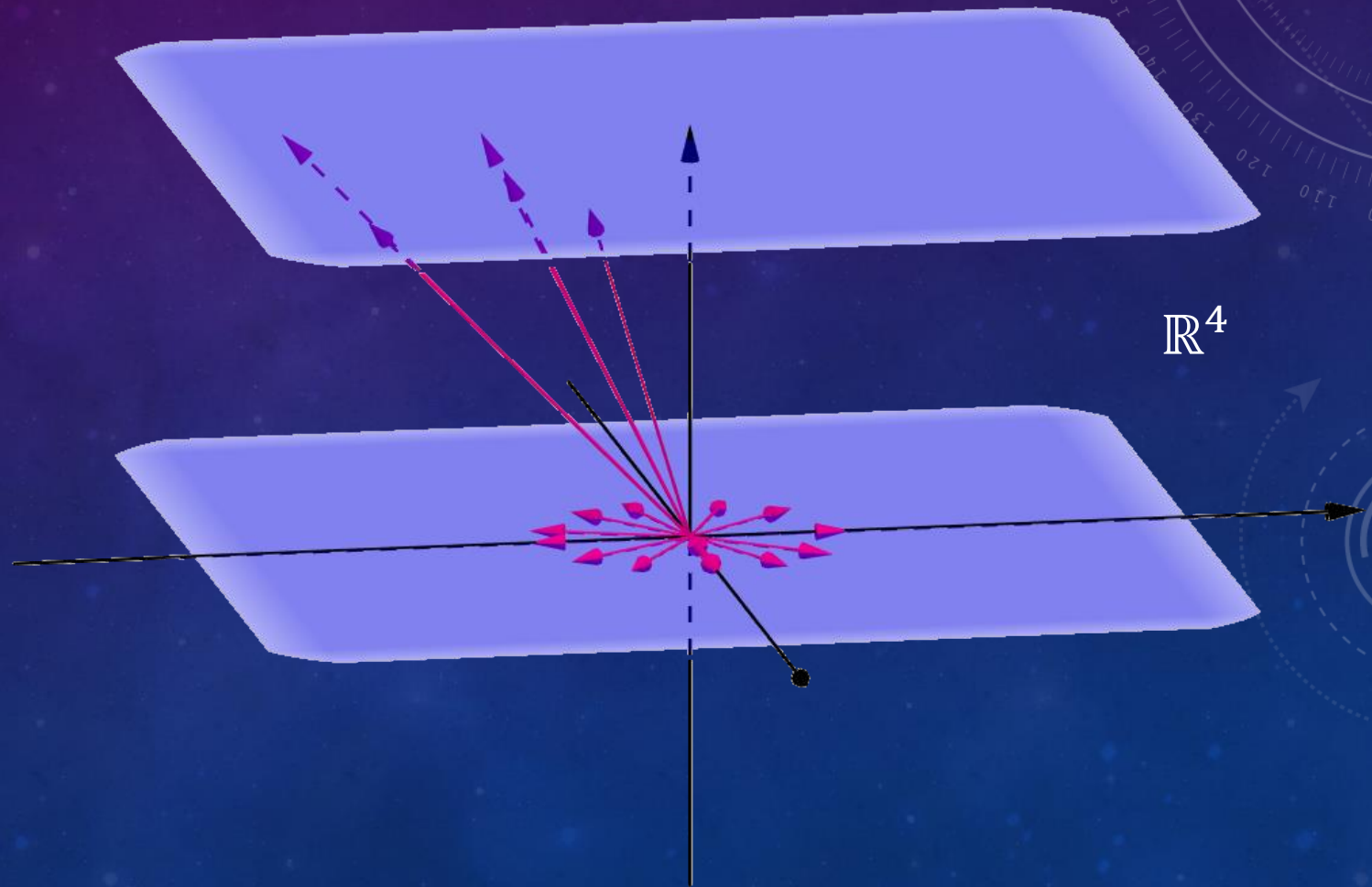
PROJECTIVE SPACE \mathbb{RP}^3

- Elements are points in $\mathbb{E}^3 \cup \mathbb{RP}^2$.



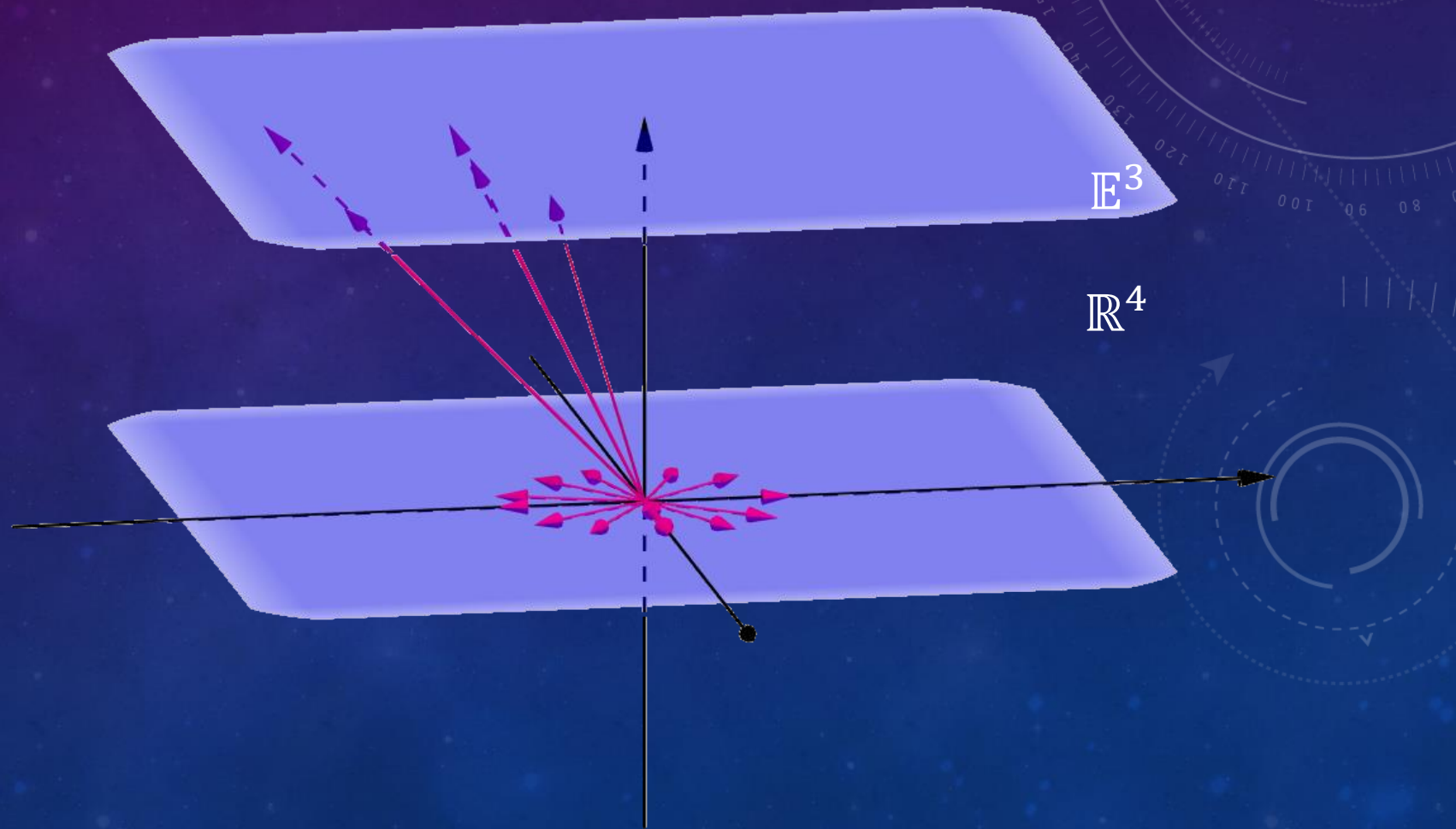
PROJECTIVE SPACE \mathbb{RP}^3

- Elements are points in $\mathbb{E}^3 \cup \mathbb{RP}^2$.



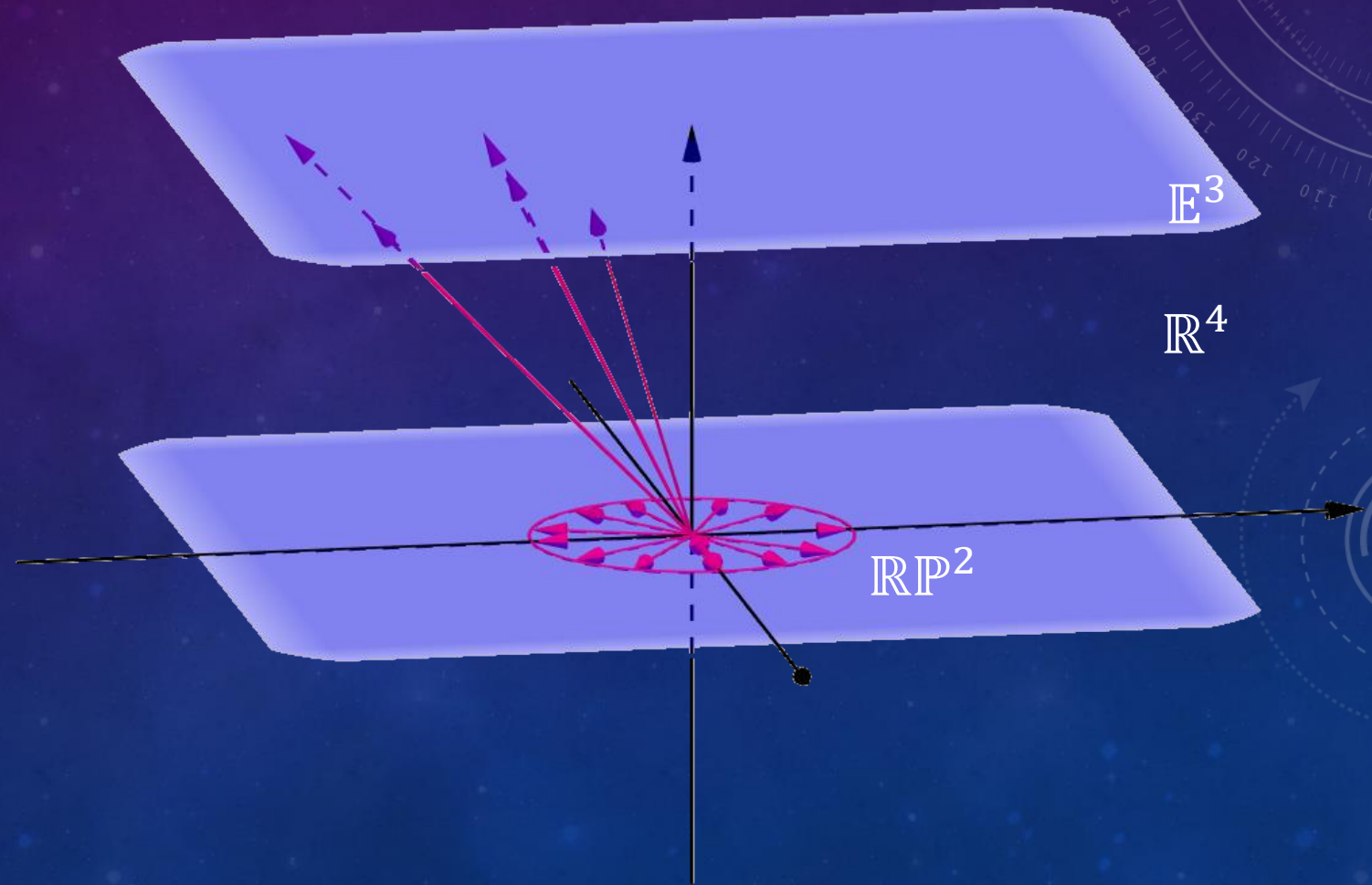
PROJECTIVE SPACE \mathbb{RP}^3

- Elements are points in $\mathbb{E}^3 \cup \mathbb{RP}^2$.



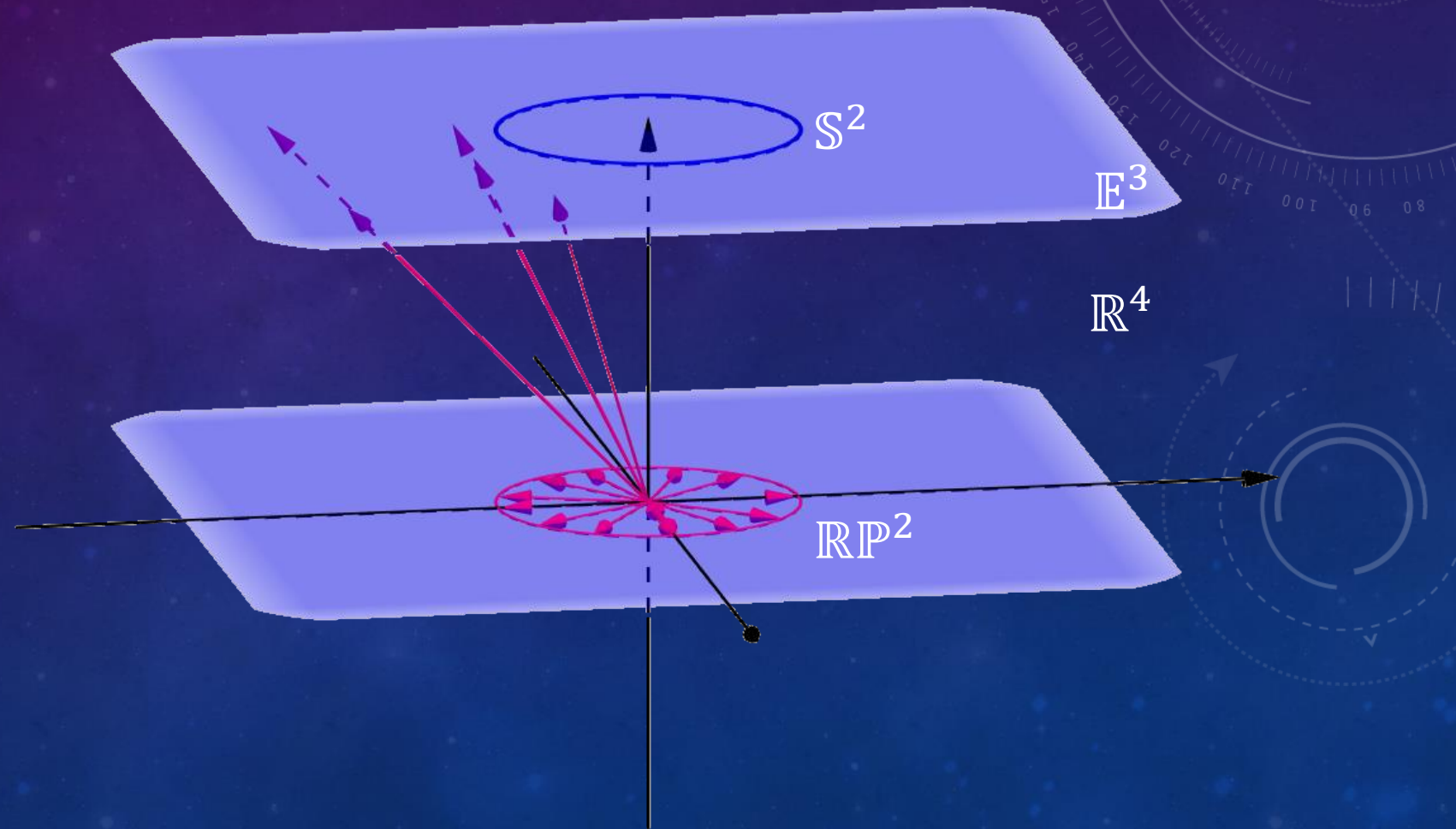
PROJECTIVE SPACE \mathbb{RP}^3

- Elements are points in $\mathbb{E}^3 \cup \mathbb{RP}^2$.



PROJECTIVE SPACE \mathbb{RP}^3

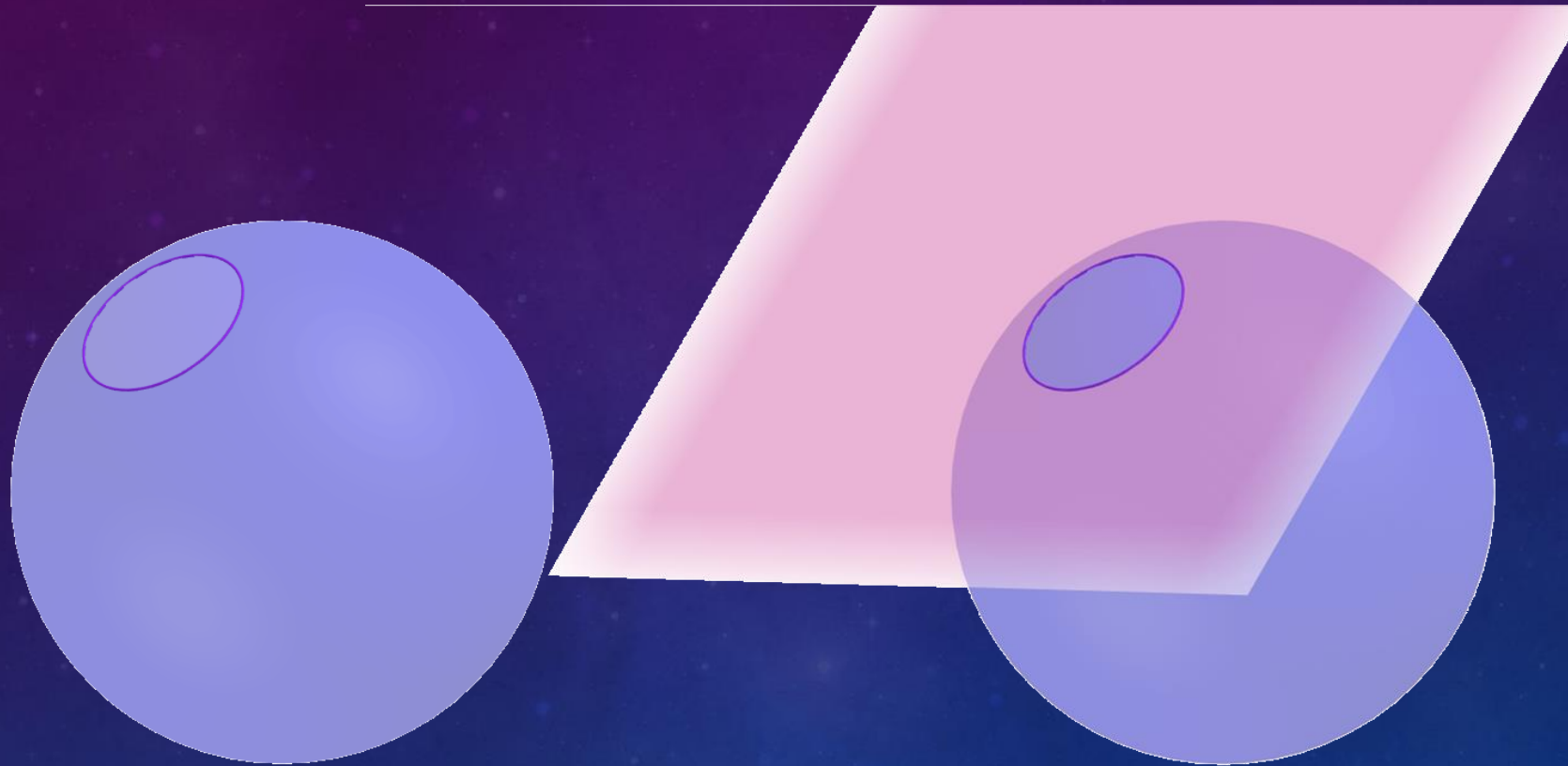
- Elements are points in $\mathbb{E}^3 \cup \mathbb{RP}^2$.
- Circle geometry is a subgeometry of projective geometry.



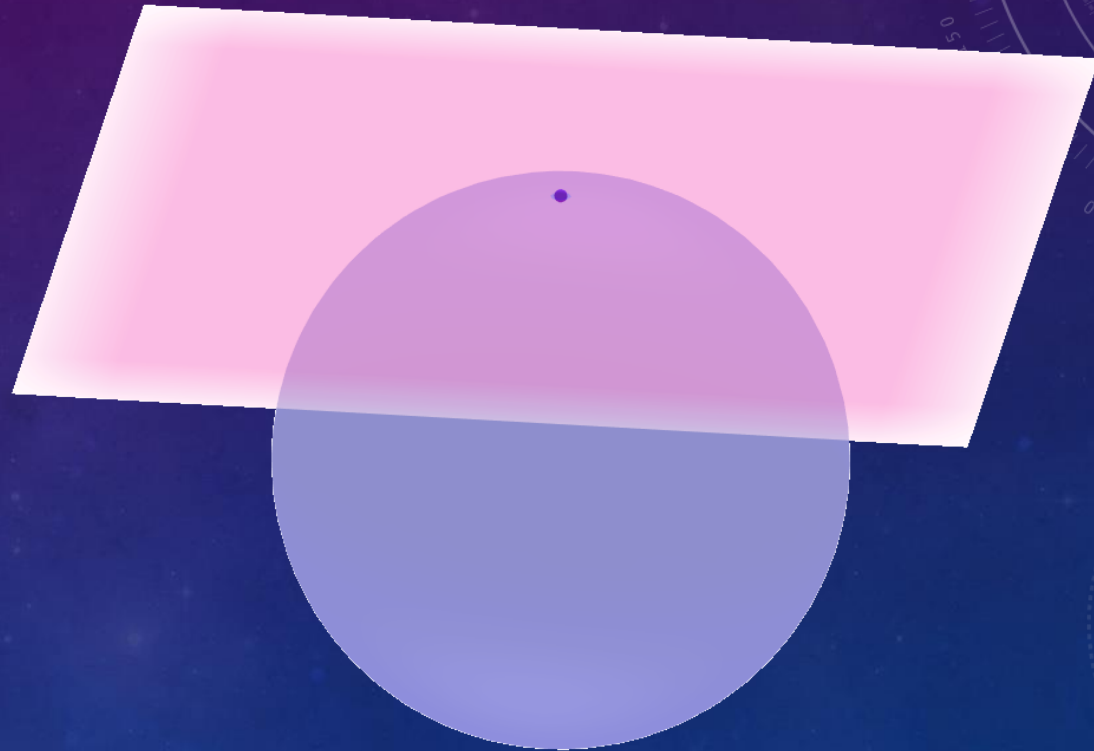
C-POINTS



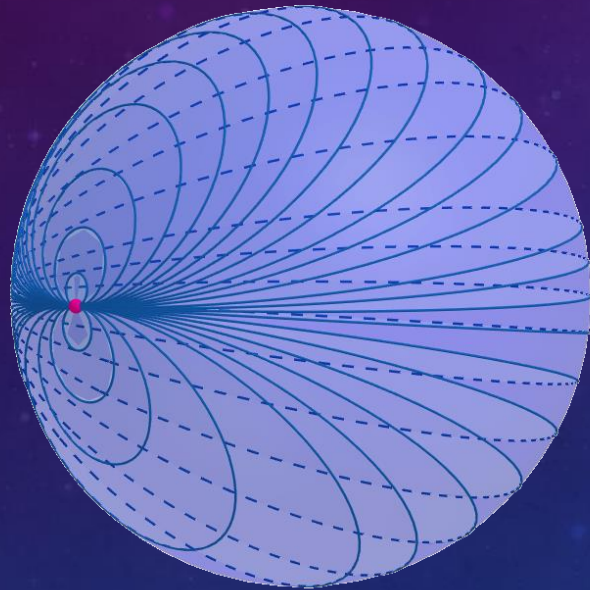
C-POINTS → PROJECTIVE PLANES



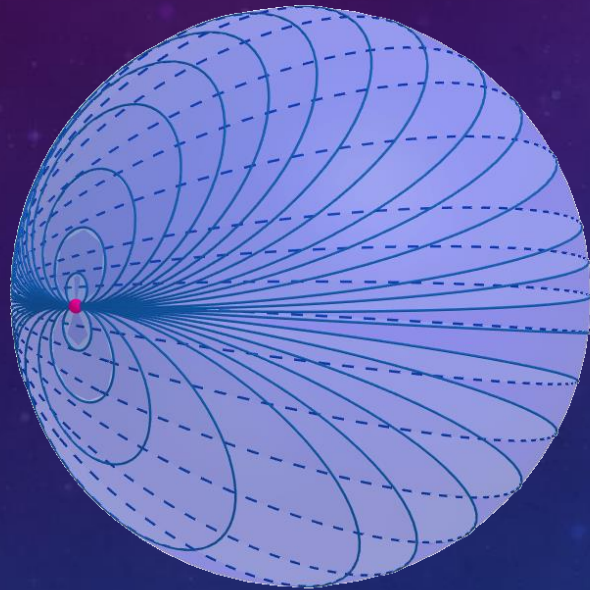
C-POINTS → PROJECTIVE PLANES



C-LINES



C-LINES

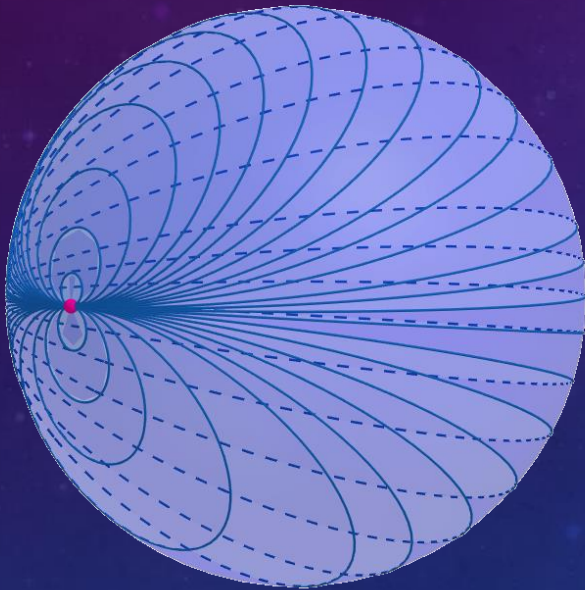


1. Intersect at one point

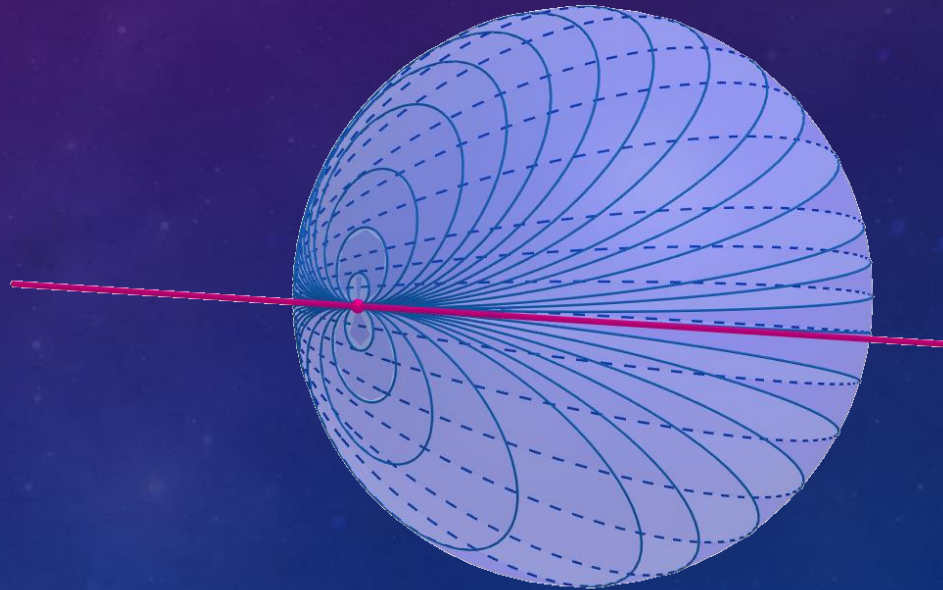
C-LINES



PROJECTIVE LINES



1. Intersect at one point

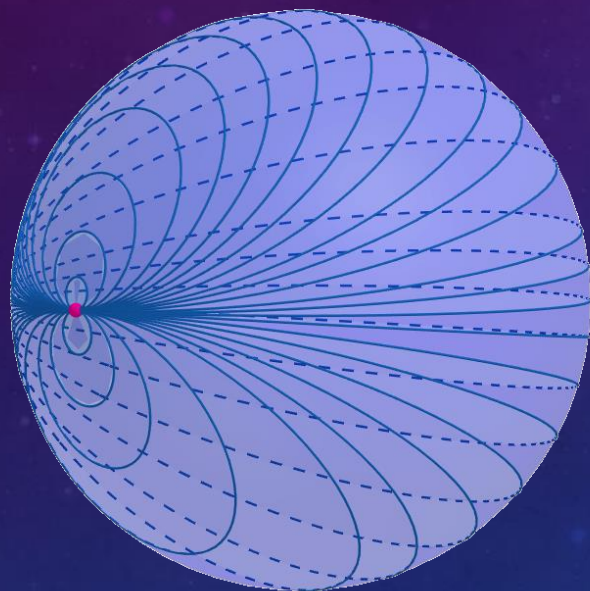


Tangent to \mathbb{S}^2 at point

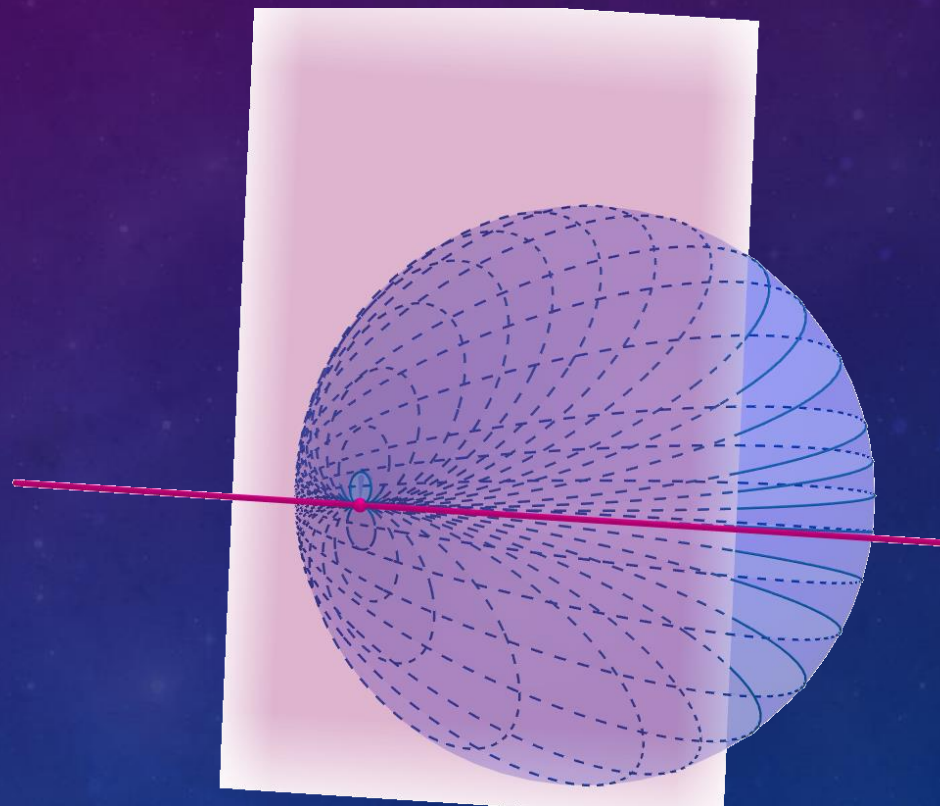
C-LINES



PROJECTIVE LINES



1. Intersect at one point

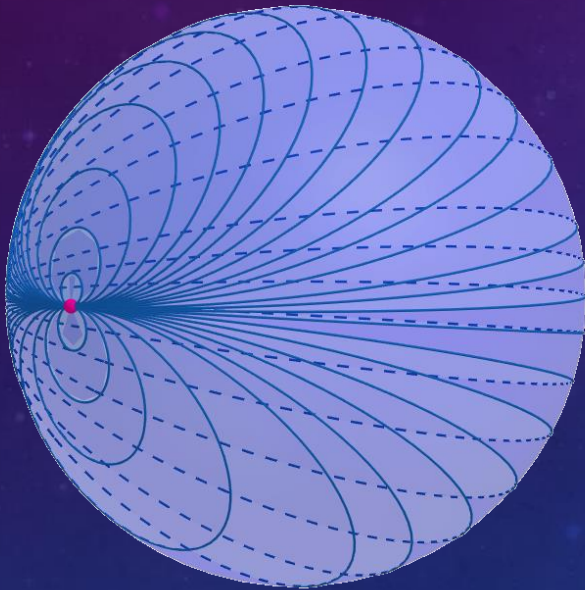


Tangent to \mathbb{S}^2 at point

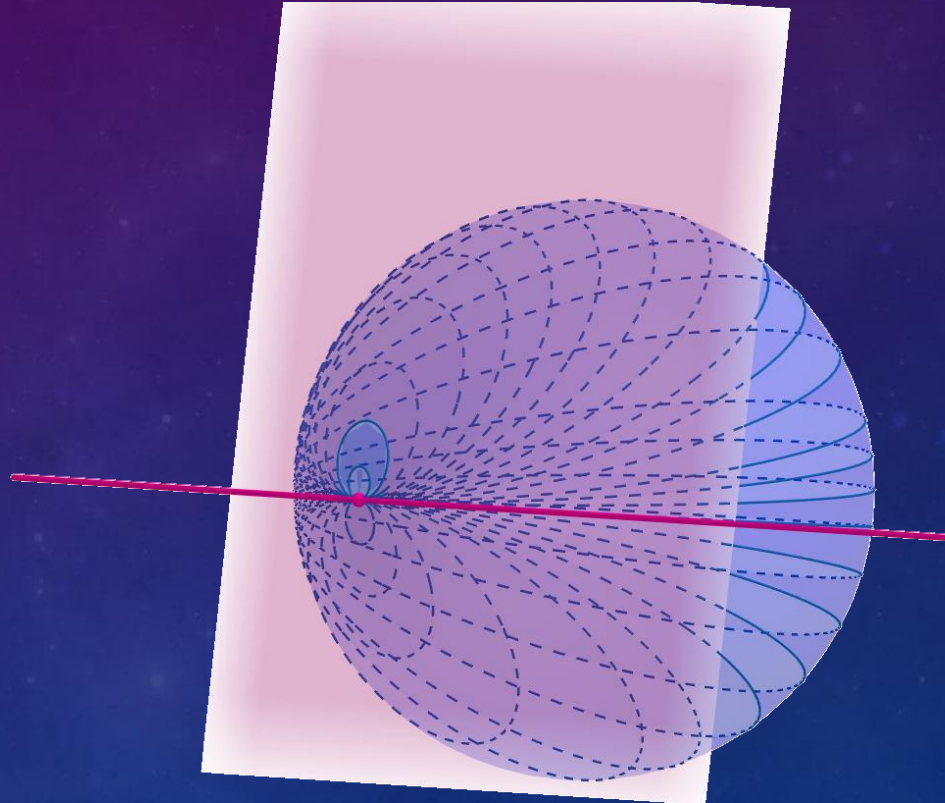
C-LINES



PROJECTIVE LINES



1. Intersect at one point

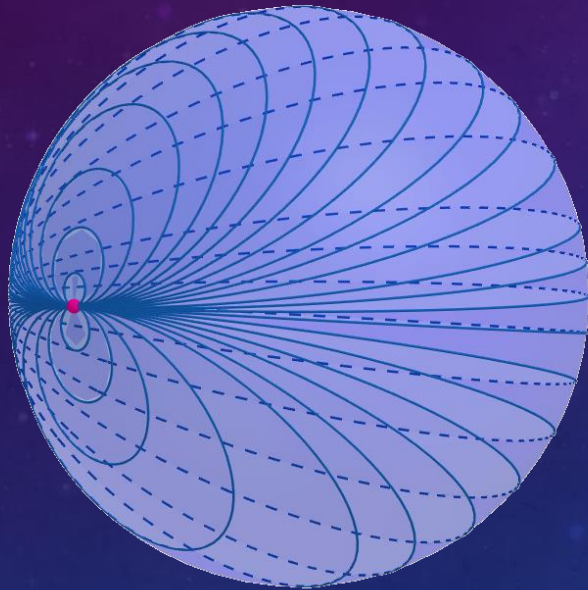


Tangent to \mathbb{S}^2 at point

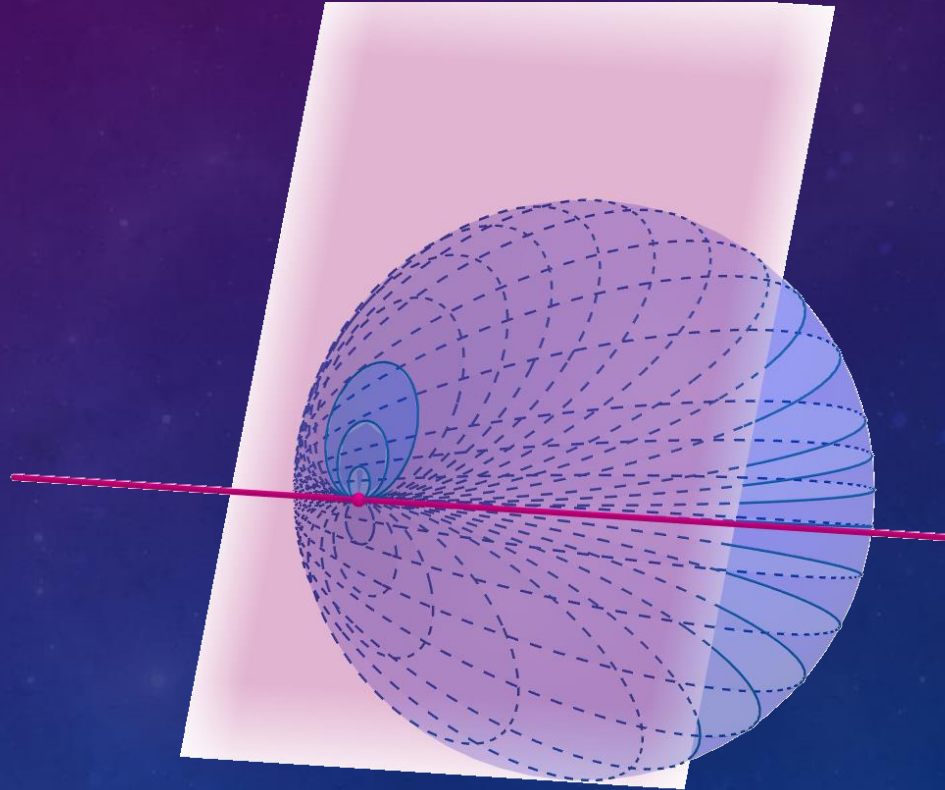
C-LINES



PROJECTIVE LINES



1. Intersect at one point

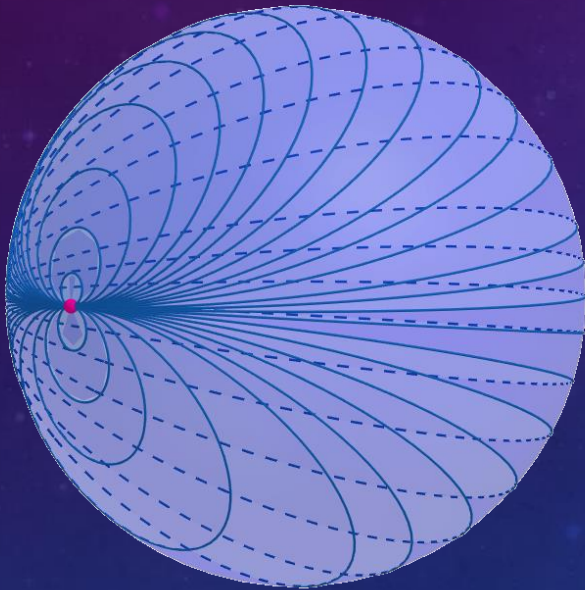


Tangent to \mathbb{S}^2 at point

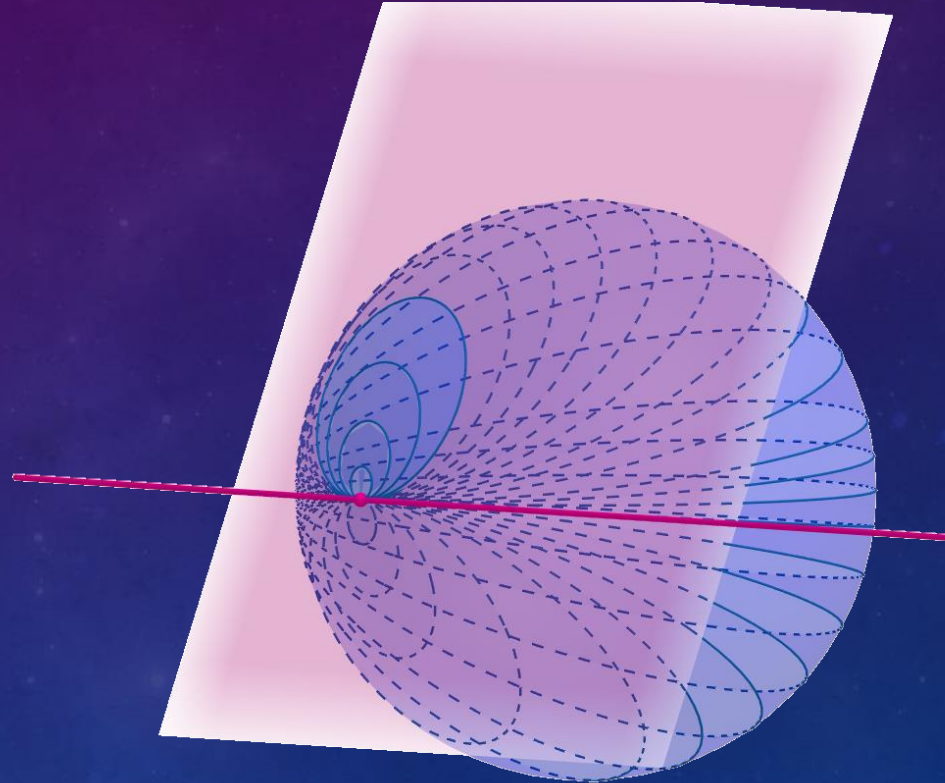
C-LINES



PROJECTIVE LINES



1. Intersect at one point

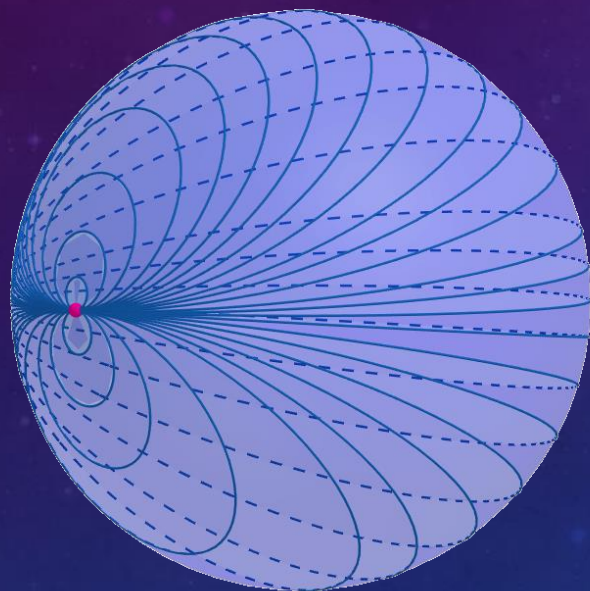


Tangent to \mathbb{S}^2 at point

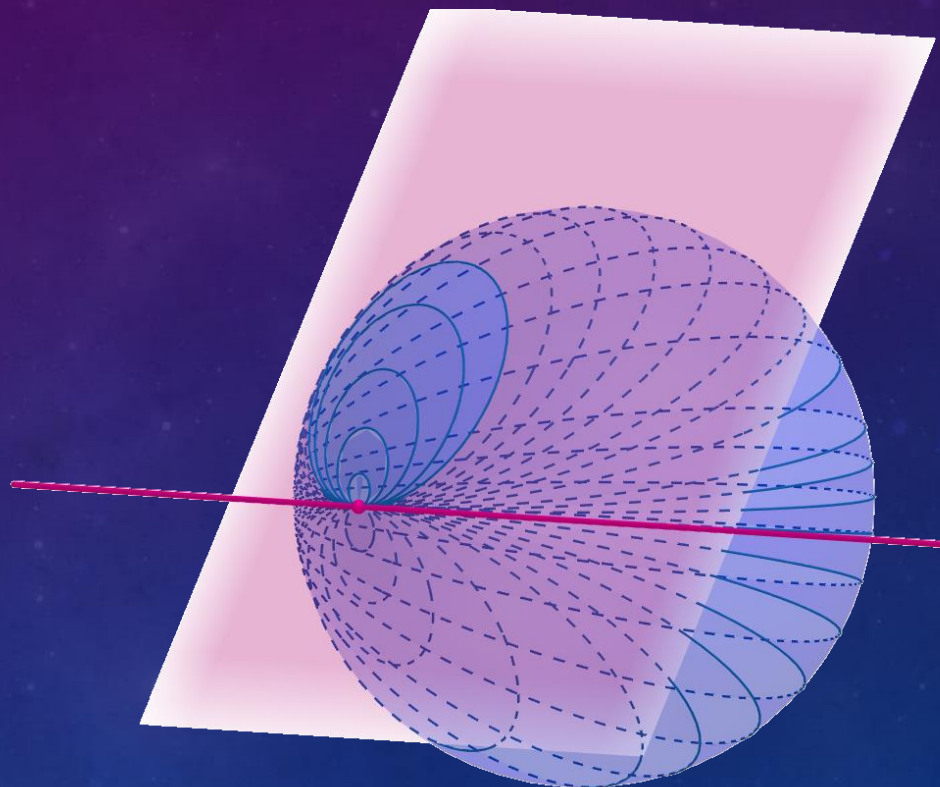
C-LINES



PROJECTIVE LINES



1. Intersect at one point

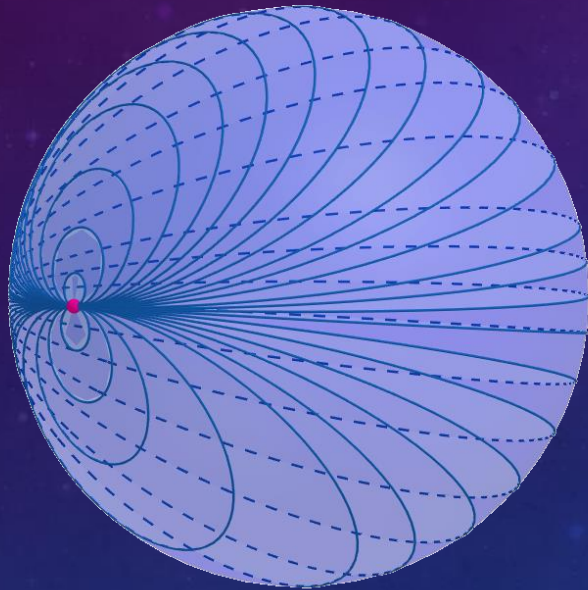


Tangent to \mathbb{S}^2 at point

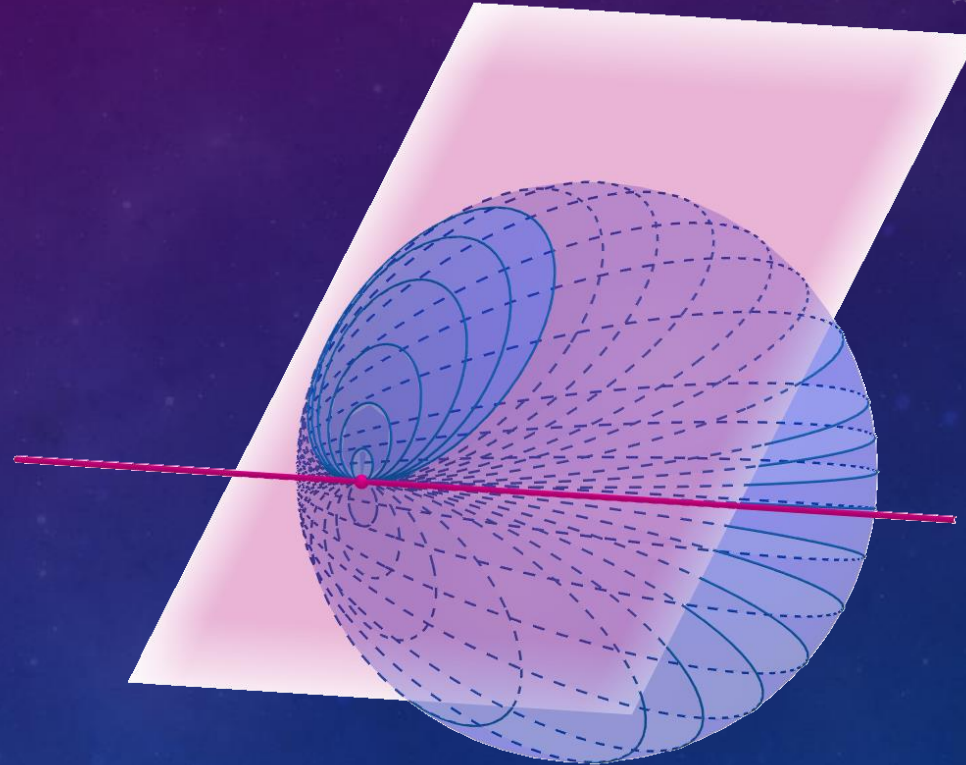
C-LINES



PROJECTIVE LINES



1. Intersect at one point

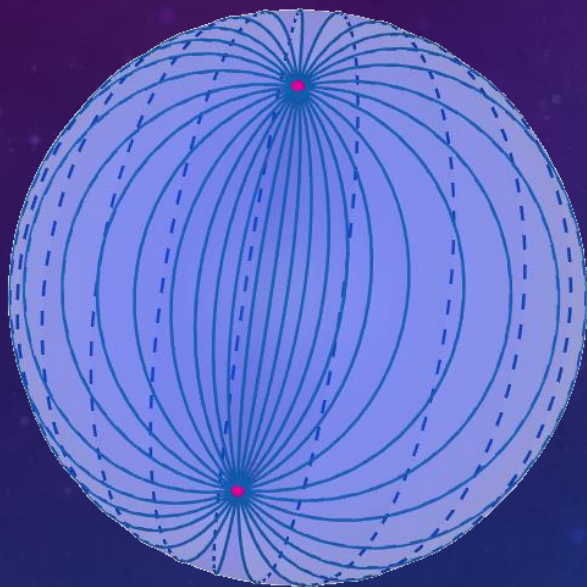


Tangent to \mathbb{S}^2 at point

C-LINES



PROJECTIVE LINES

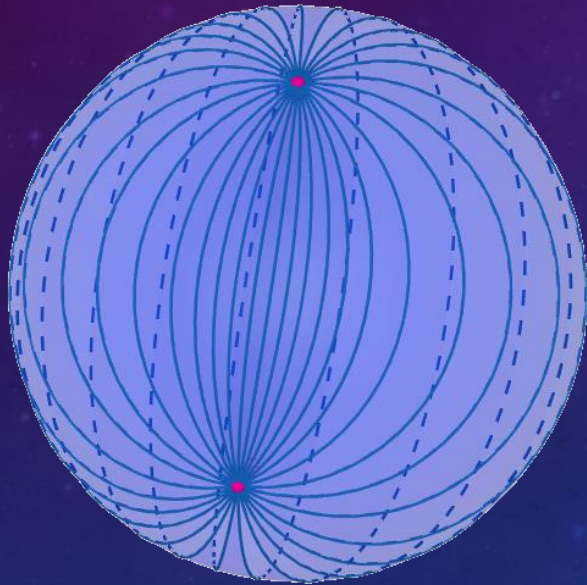


2. Intersect at two points

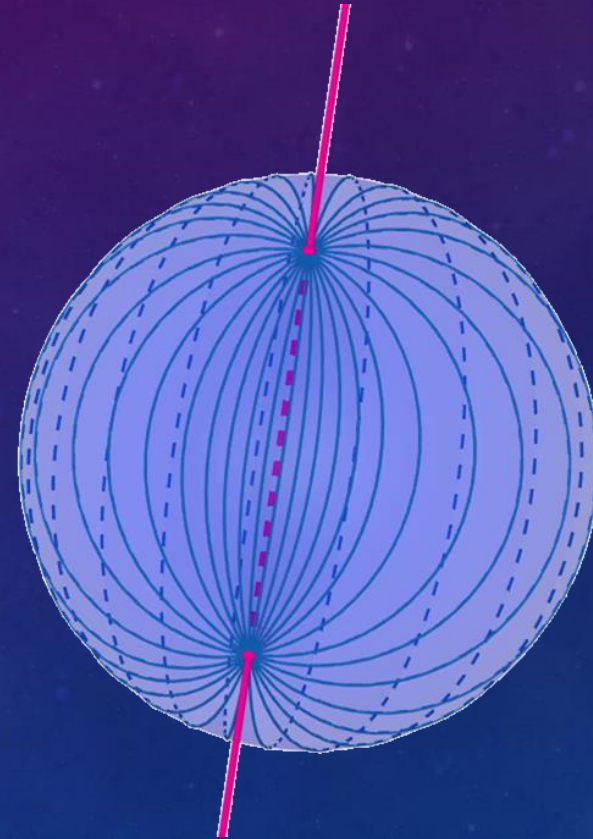
C-LINES



PROJECTIVE LINES



2. Intersect at two points

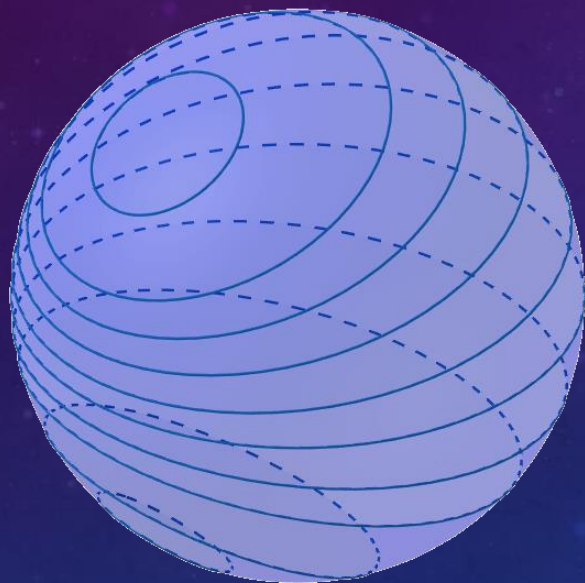


Intersects S^2 at two points

C-LINES



PROJECTIVE LINES

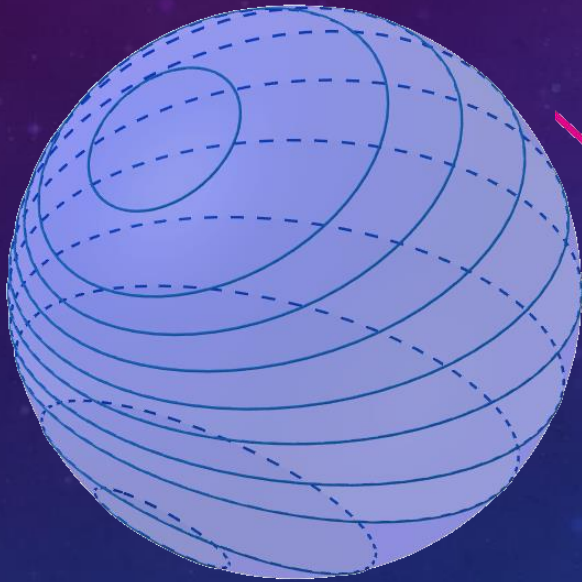


3. Don't intersect

C-LINES



PROJECTIVE LINES

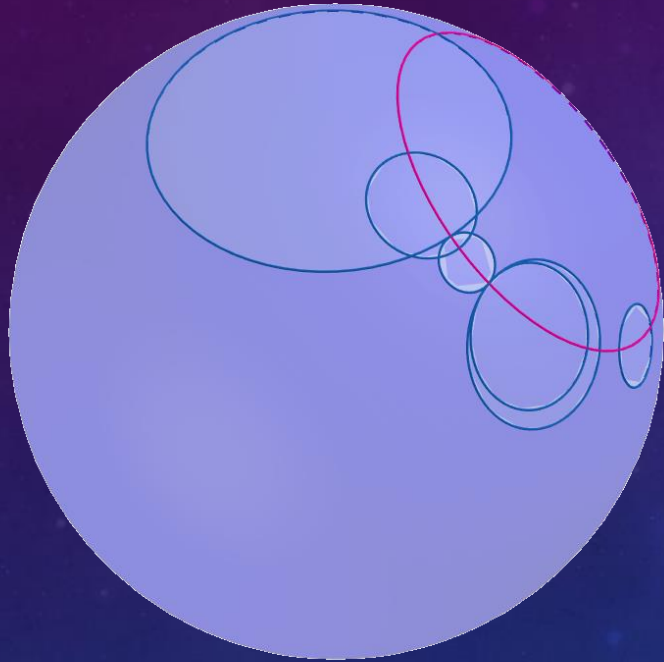


3. Don't intersect

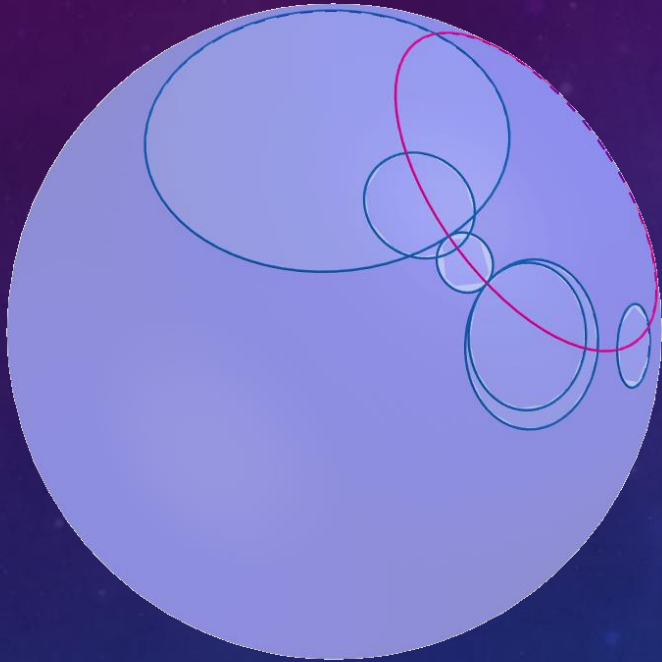


Doesn't intersect \mathbb{S}^2 .

C-PLANES



C-PLANES

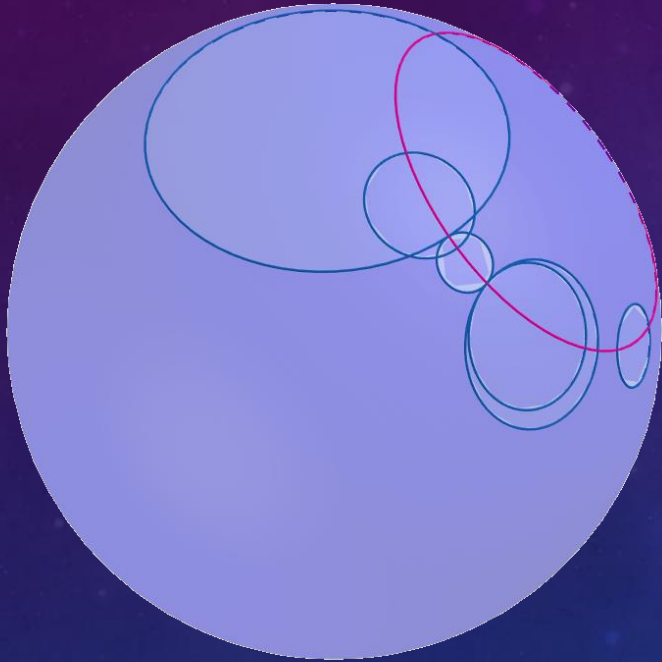


1. All circles orthogonal to generating circle

C-PLANES



PROJECTIVE POINTS



1. All circles
orthogonal to
generating circle

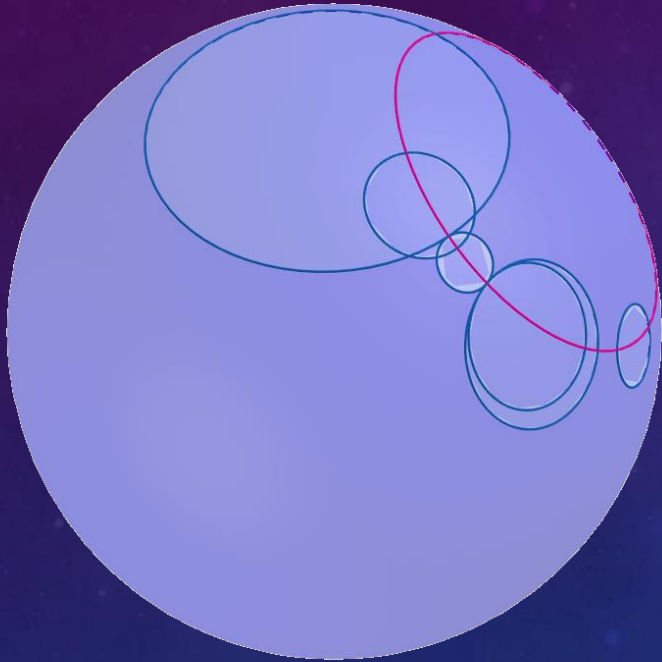


Point outside sphere

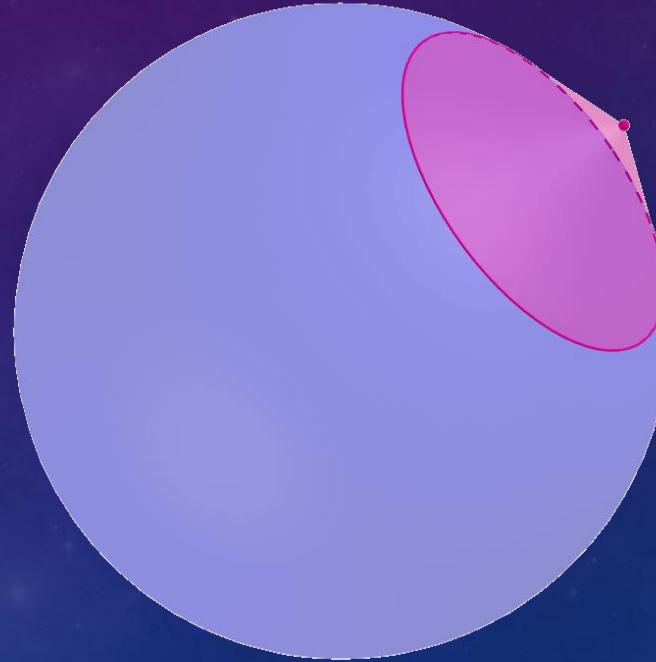
C-PLANES



PROJECTIVE POINTS



1. All circles
orthogonal to
generating circle

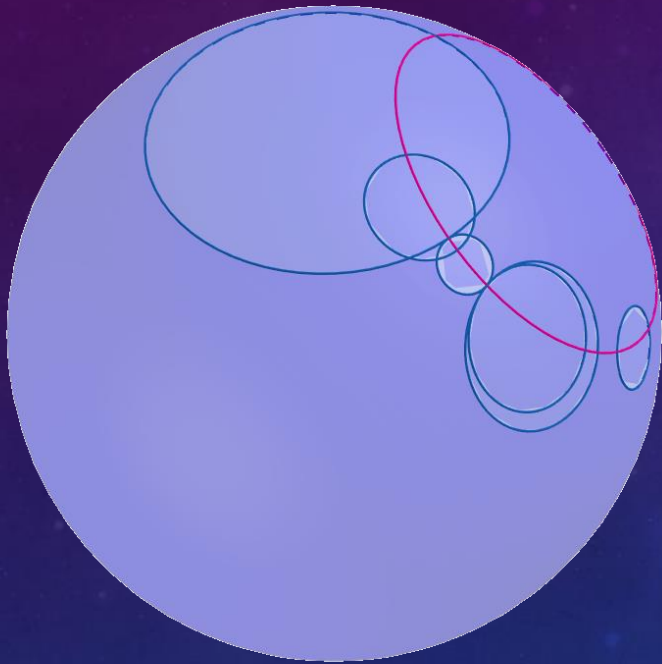


Point outside sphere

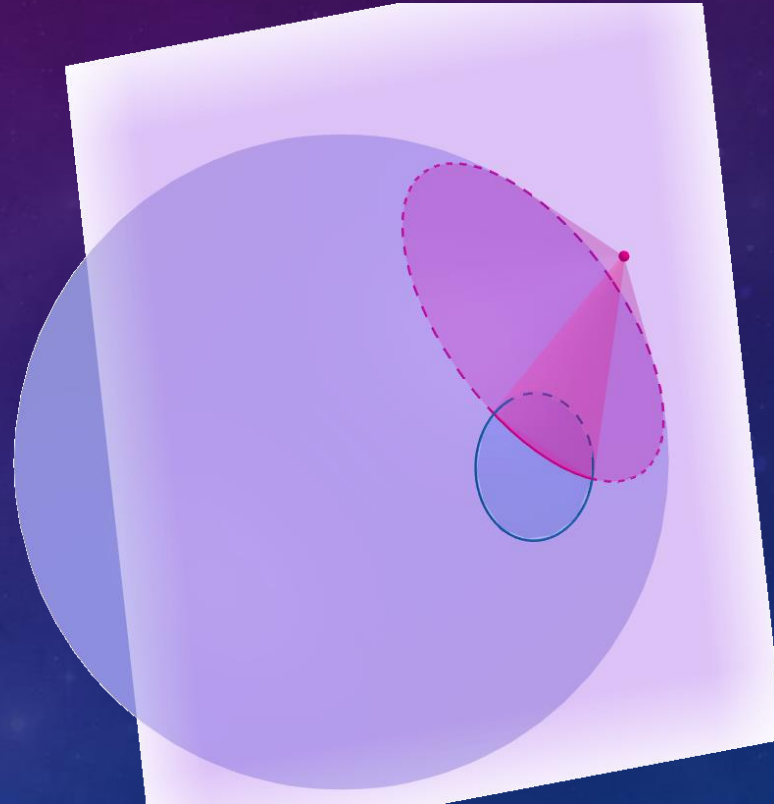
C-PLANES



PROJECTIVE POINTS

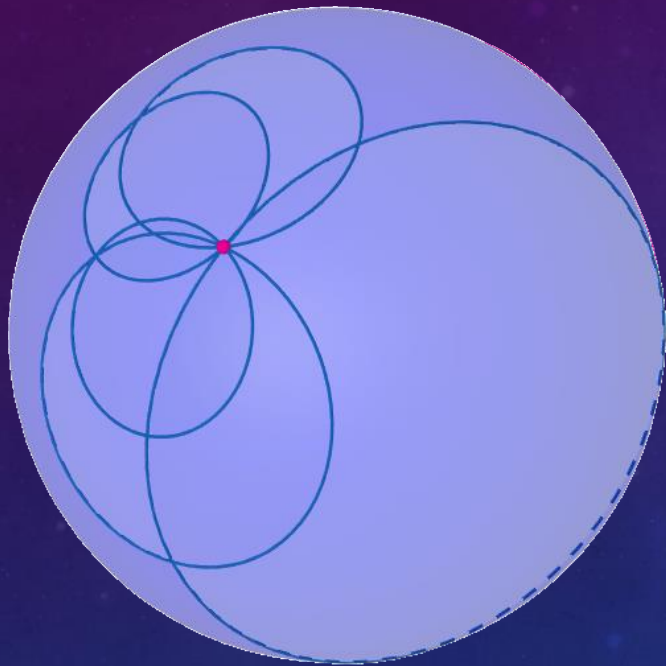


1. All circles
orthogonal to
generating circle



Point outside sphere

C-PLANES \leftrightarrow PROJECTIVE POINTS

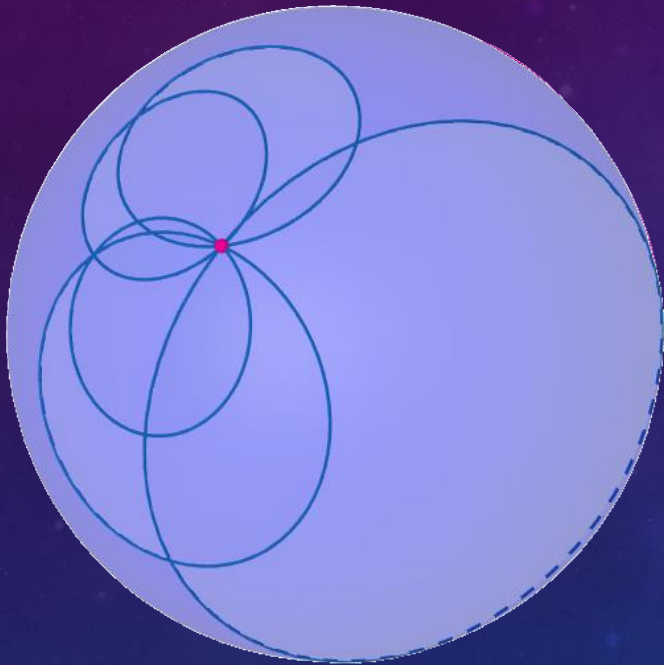


2. All circles
through generating
“circle”

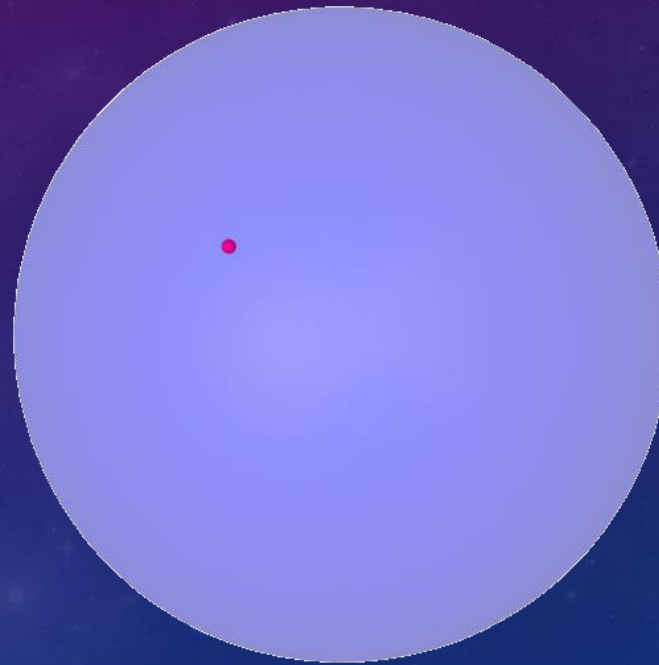
C-PLANES



PROJECTIVE POINTS

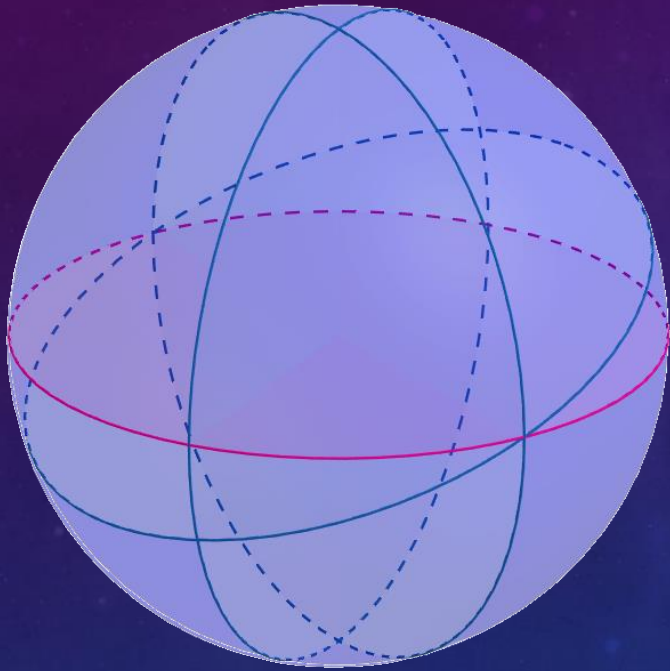


2. All circles
through generating
“circle”



Point on sphere

C-PLANES \leftrightarrow PROJECTIVE POINTS

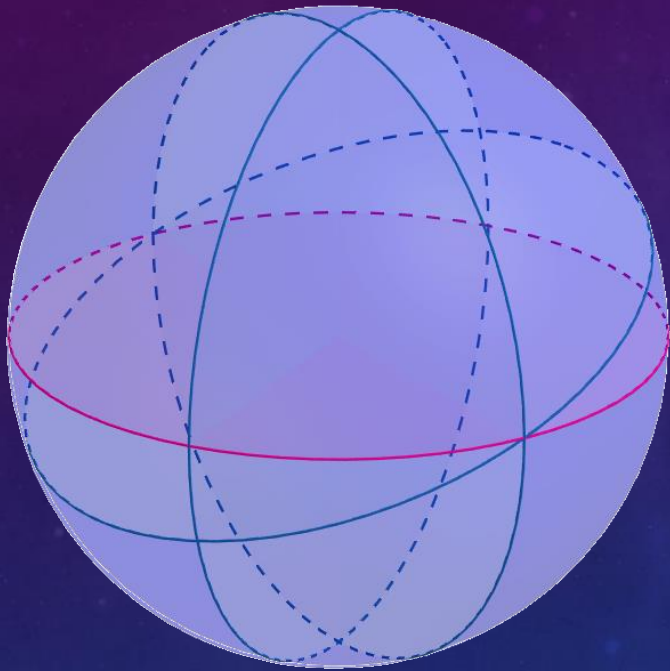


3. All circles
through antipodal
points on
generating circle

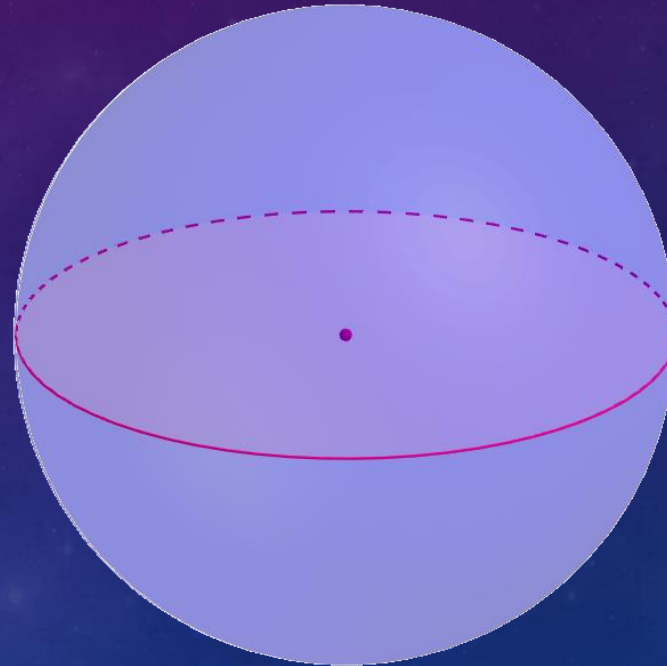
C-PLANES



PROJECTIVE POINTS



3. All circles
through antipodal
points on
generating circle

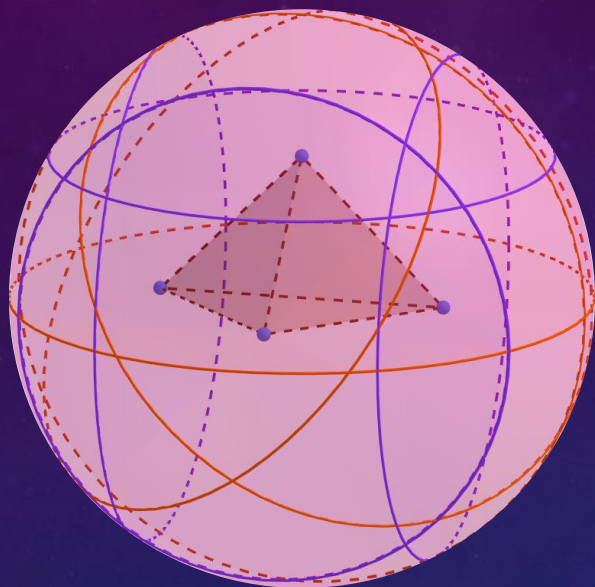


Point inside sphere

C-POLYHEDRA



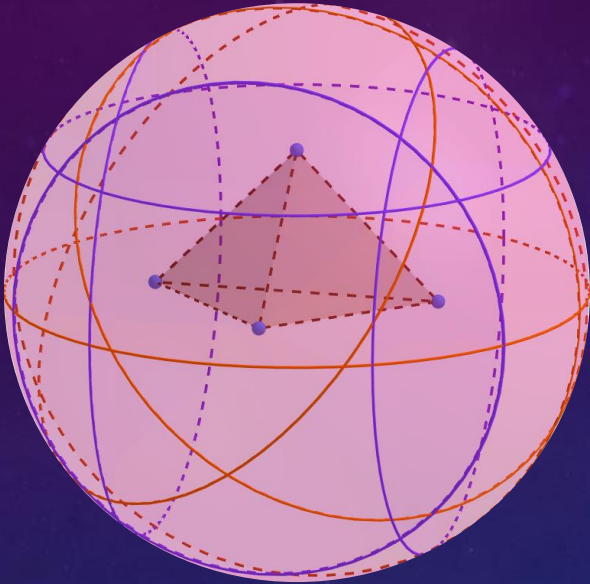
PROJECTIVE POLYHEDRA



C-POLYHEDRA



PROJECTIVE POLYHEDRA

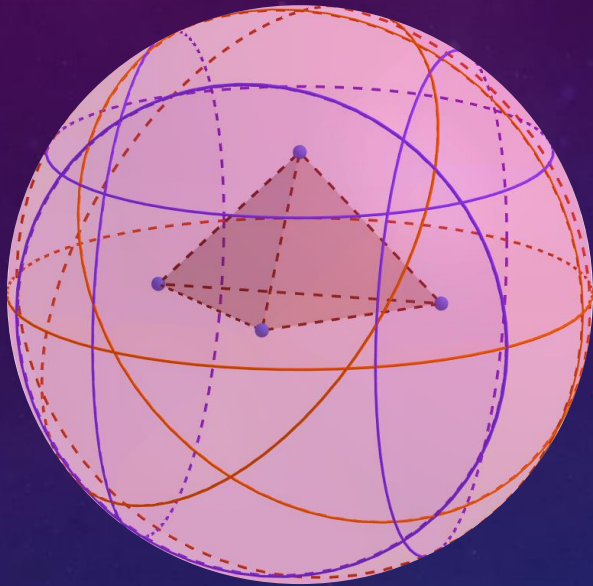


- ▲ Hyperbolic polyhedra :
- Andre'ev ('70)
 - Rivin & Hodgson ('93)

C-POLYHEDRA

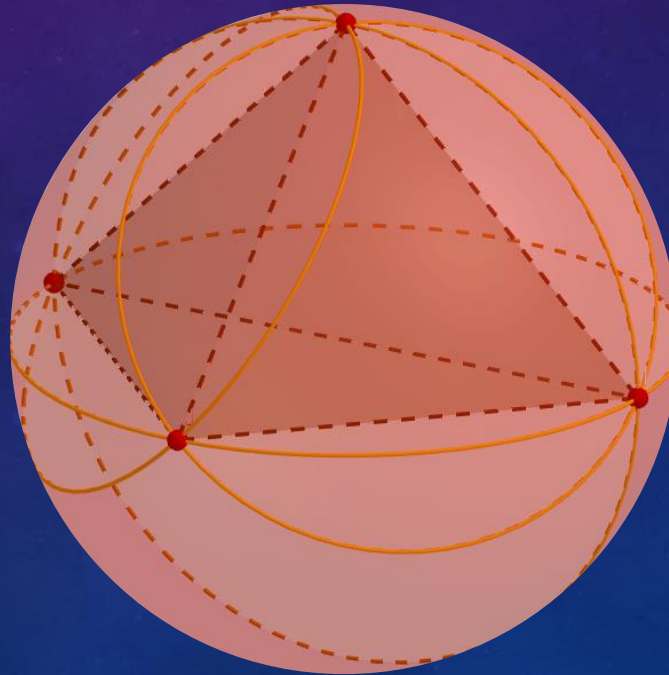


PROJECTIVE POLYHEDRA



- ▲ Hyperbolic polyhedra :
- Andre'ev ('70)
 - Rivin & Hodgson ('93)

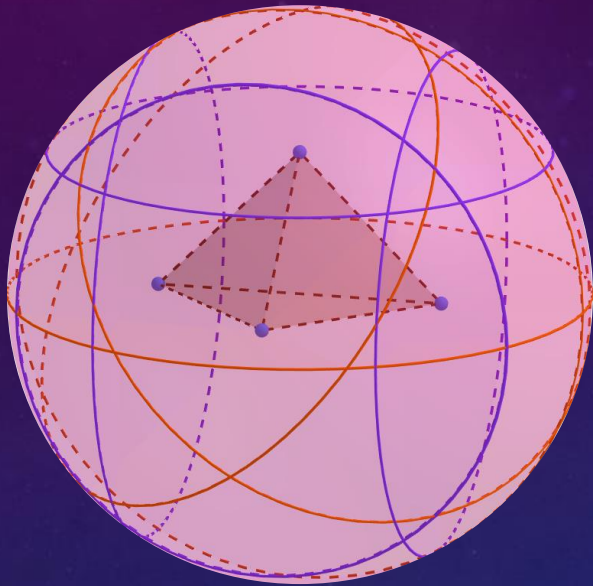
Ideal Polyhedra:
Rivin ('96) ▼



C-POLYHEDRA

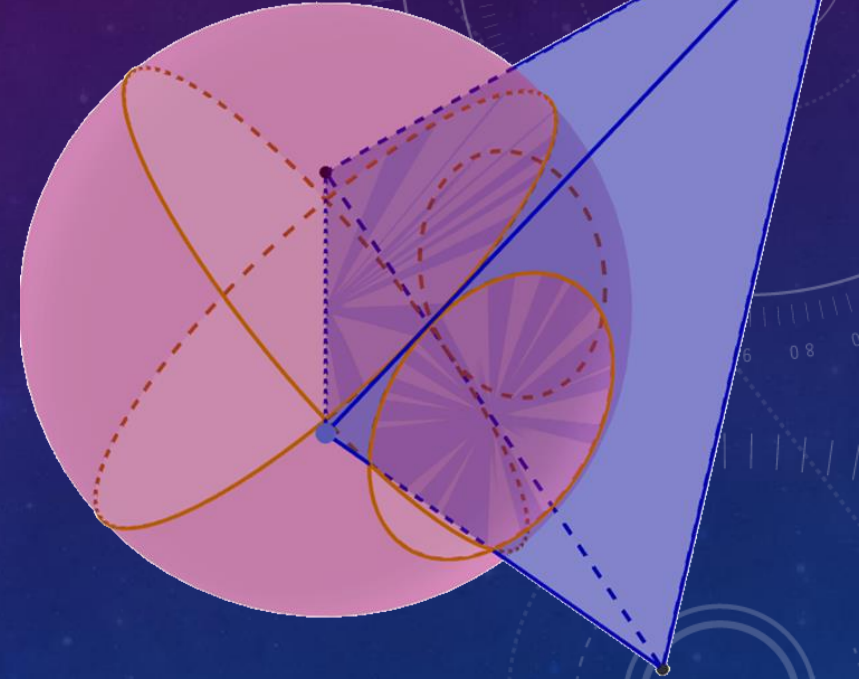
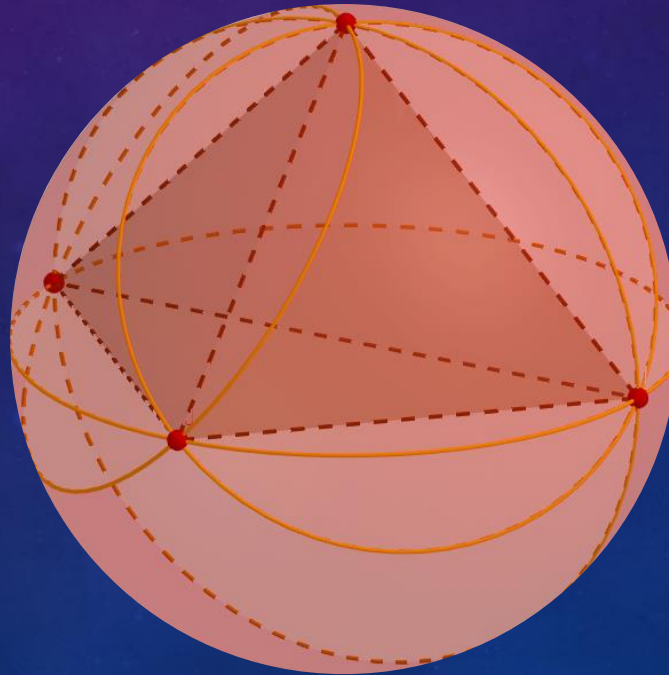


PROJECTIVE POLYHEDRA



- ▲ Hyperbolic polyhedra :
- Andre'ev ('70)
 - Rivin & Hodgson ('93)

Ideal Polyhedra:
Rivin ('96) ▼



- ▲ Hyperideal polyhedra :
- Bao & Bonahon ('02)
 - Bowers, Bowers, Pratt ('17)



“ALL GEOMETRY IS
PROJECTIVE GEOMETRY.”
ARTHUR CAYLEY



“ALL GEOMETRY IS
PROJECTIVE GEOMETRY.”

ARTHUR CAYLEY

THANK YOU!

DUALITY



DUALITY

- c-points \leftrightarrow projective points

DUALITY

- c-points \leftrightarrow projective points
- c-lines \leftrightarrow projective lines

DUALITY

- c-points \leftrightarrow projective points
- c-lines \leftrightarrow projective lines
- c-planes \rightarrow projective planes