A CORRESPONDENCE BETWEEN PROJECTIVE GEOMETRY AND CIRCLE GEOMETRY

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FLASH TALK

SPRING 2019

FLORIDA STATE UNIVERSITY
Mathematics

CIRCLE SPACE $\mathcal{C}(\mathbb{S}^2)$

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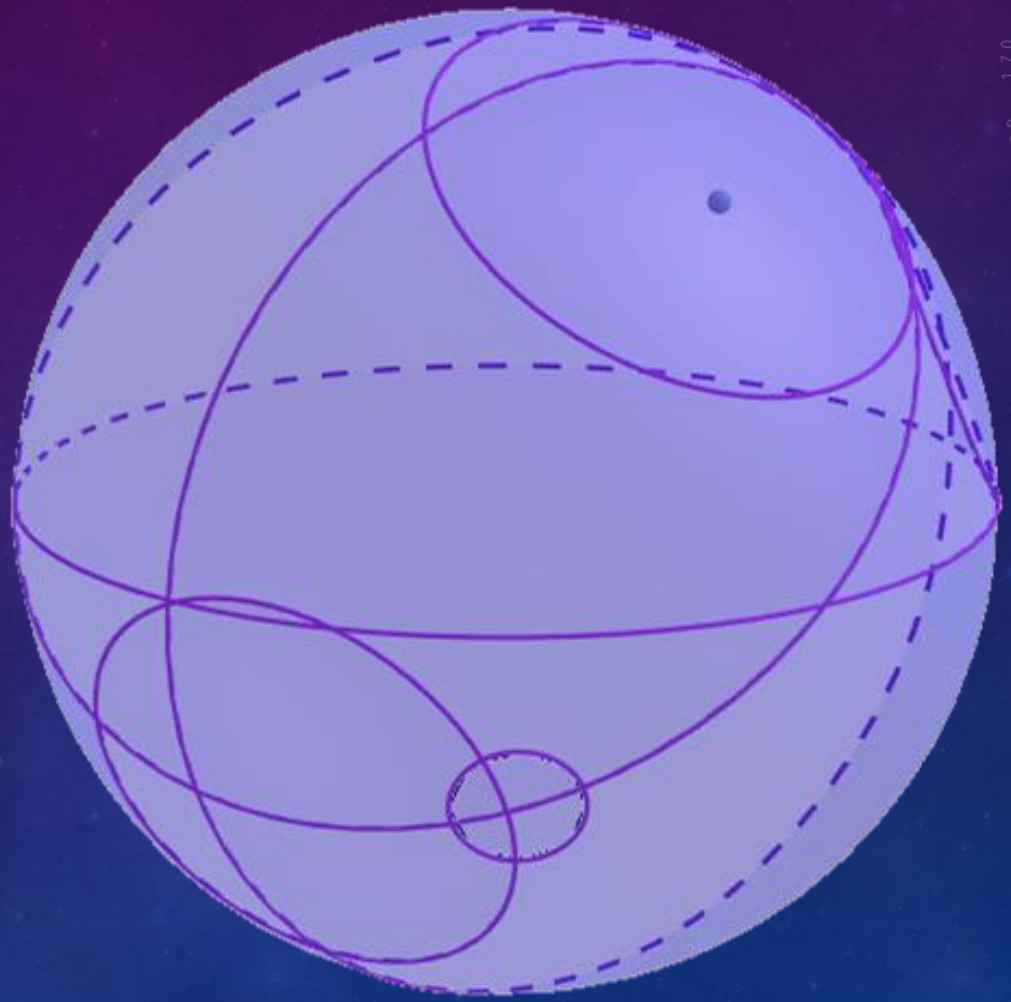
- Elements are circles of radius $0 \leq r \leq \pi$ in $\mathbb{S}^2$.



PROJECTIVE SPACE $\mathbb{RP}^3$

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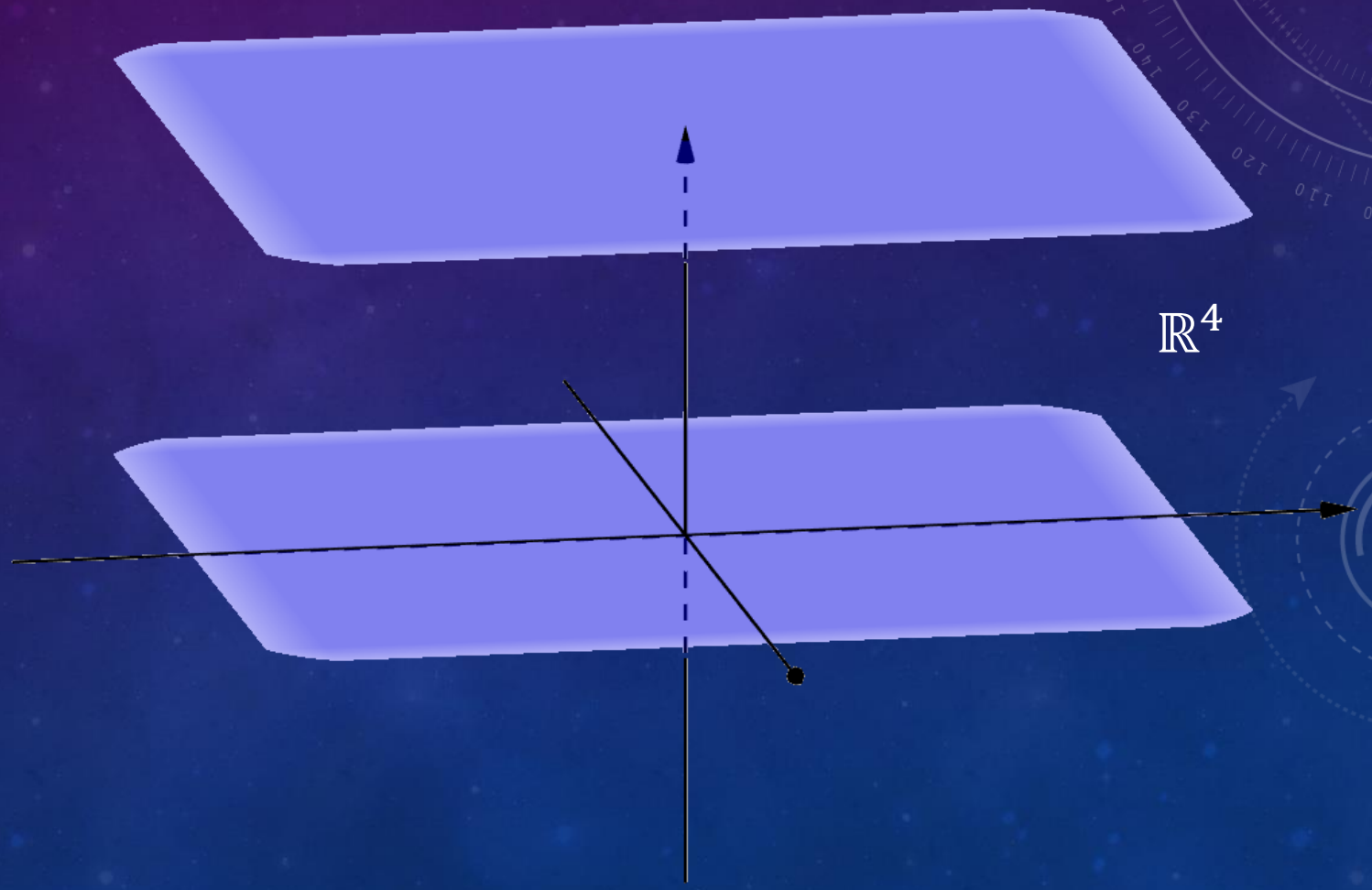

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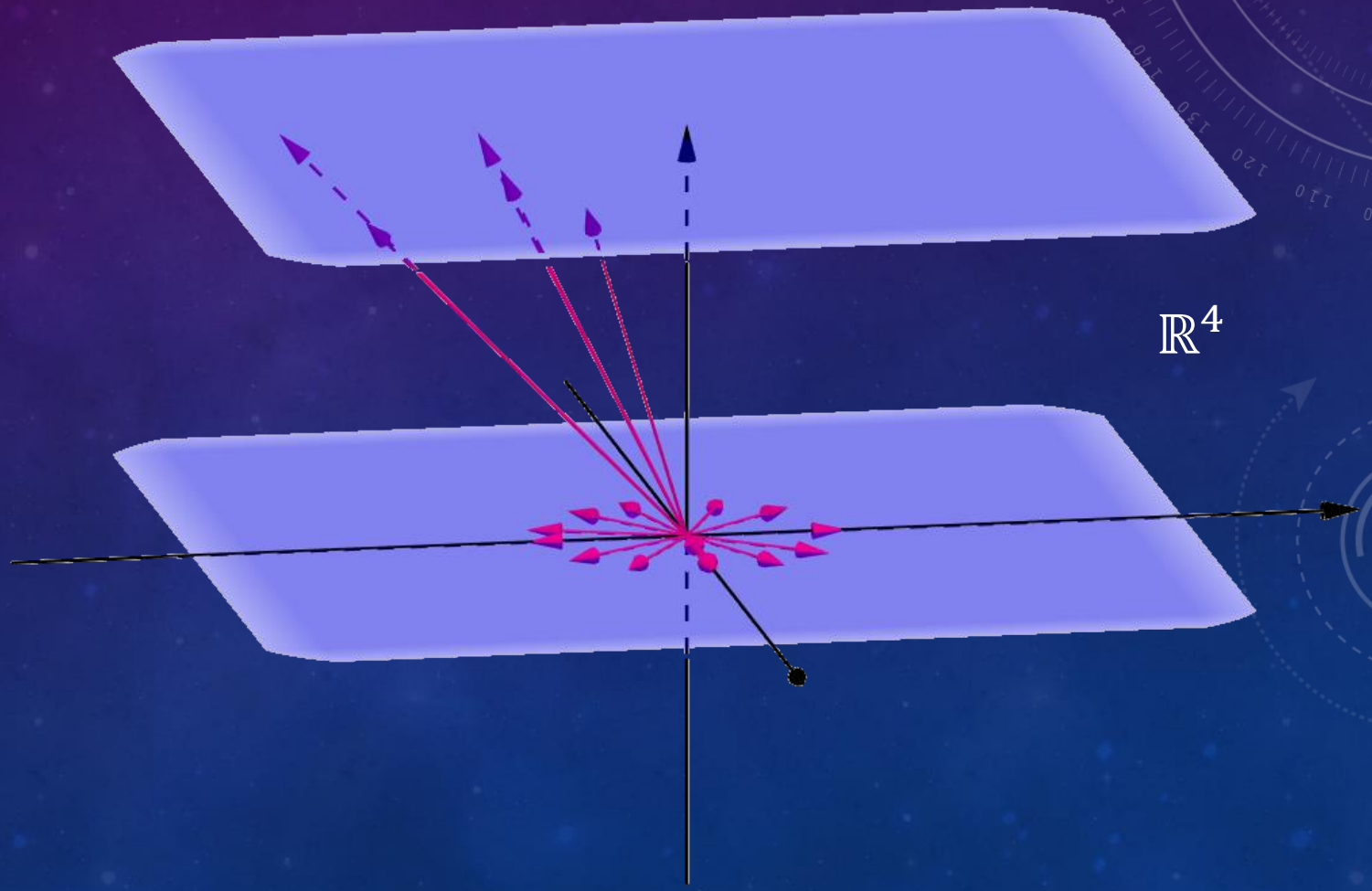

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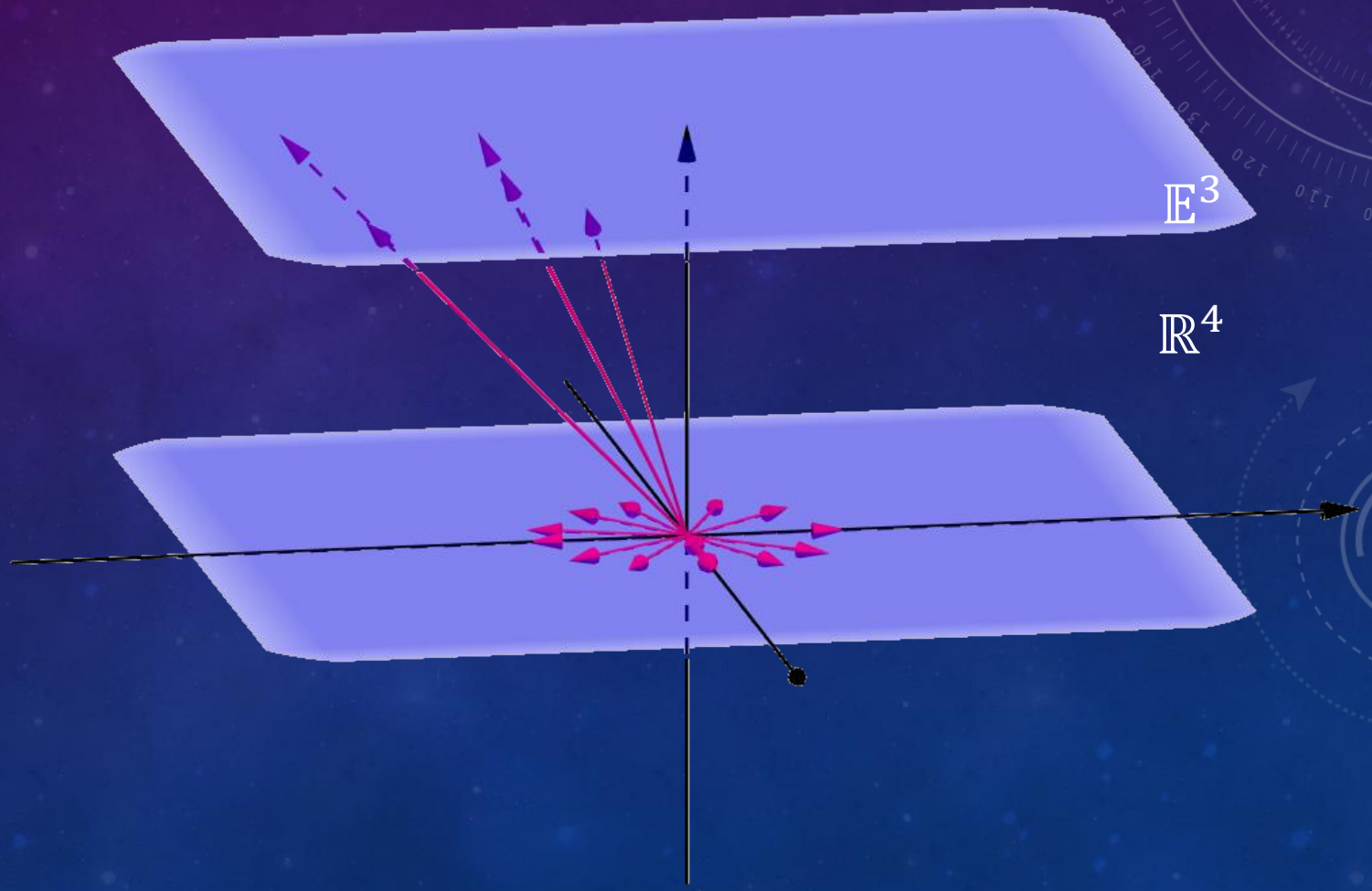

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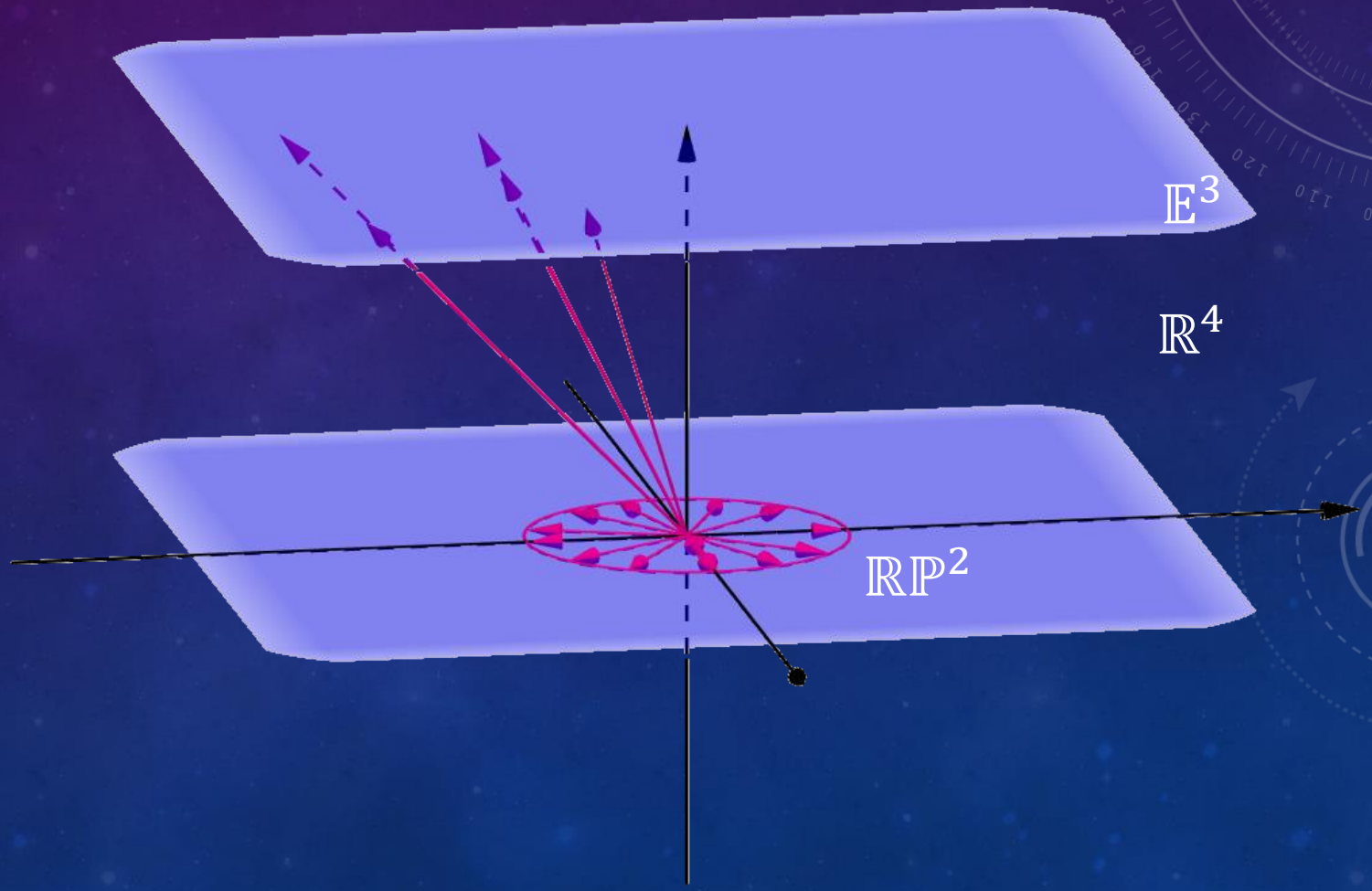
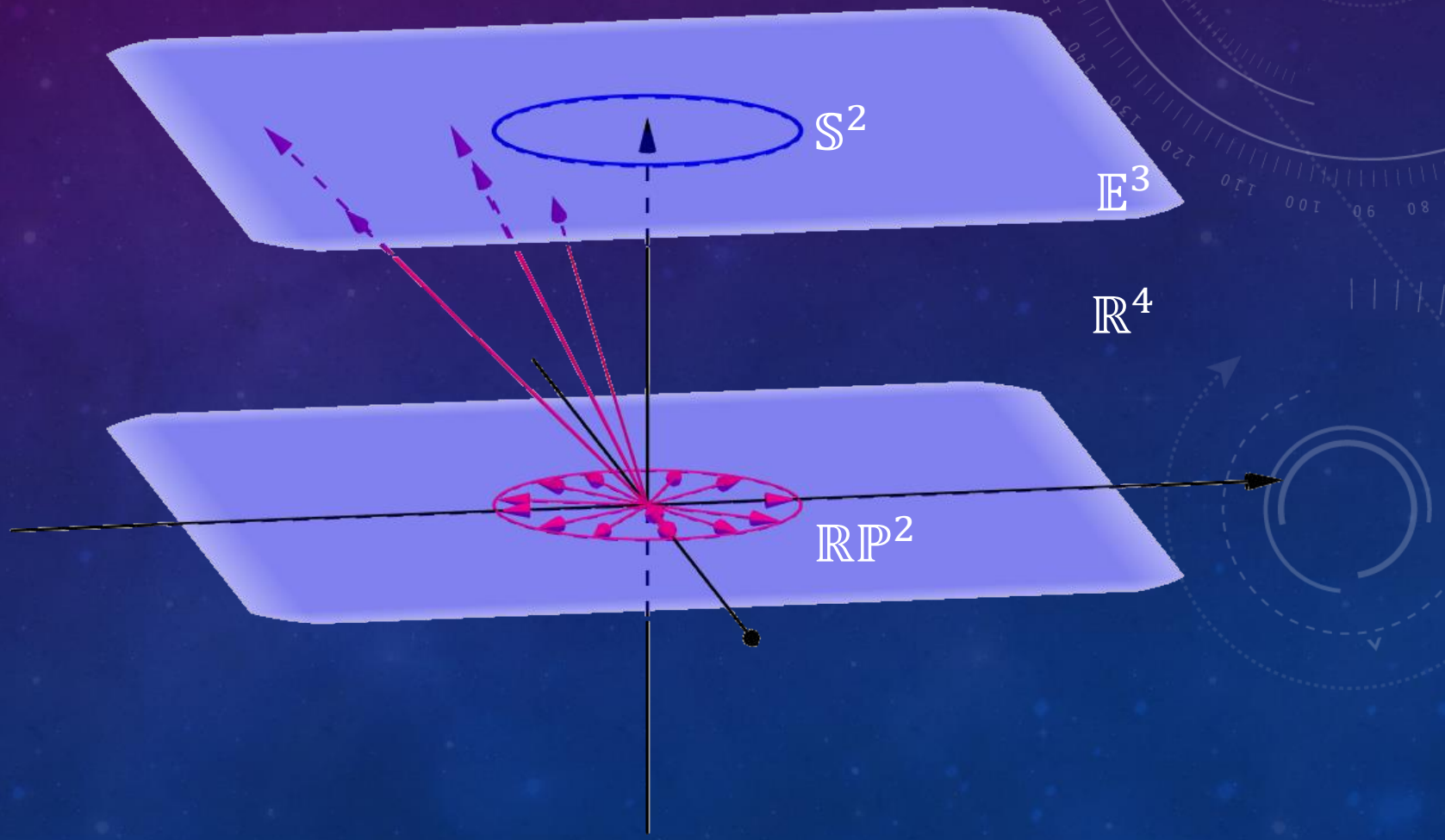


PROJECTIVE SPACE $\mathbb{RP}^3$

- Elements are points in $\mathbb{E}^3 \cup \mathbb{RP}^2$.
- Circle geometry is a subgeometry of projective geometry.



C-POINTS

C-POINTS → PROJECTIVE PLANES

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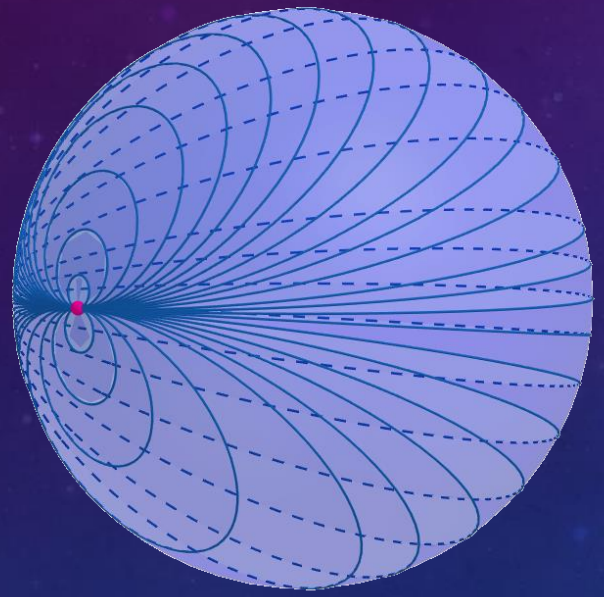
C-LINES



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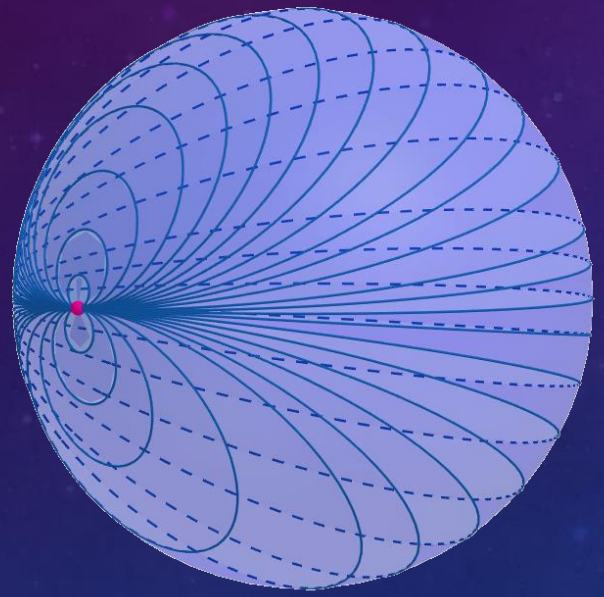
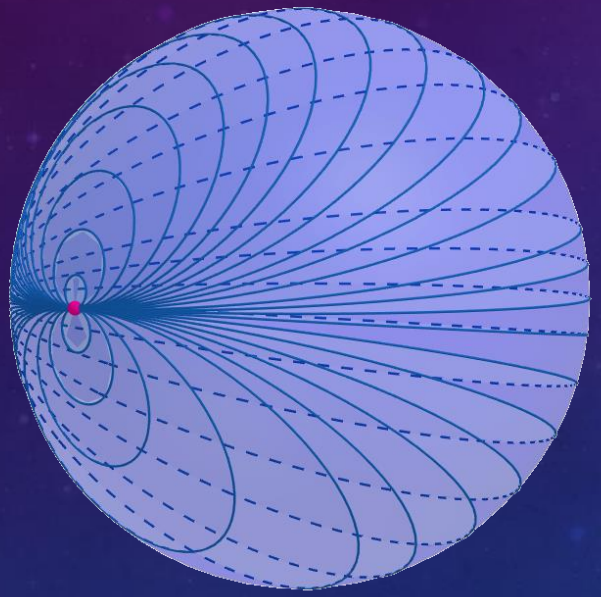


1. Intersect at one point

C-LINES ↔ PROJECTIVE LINES



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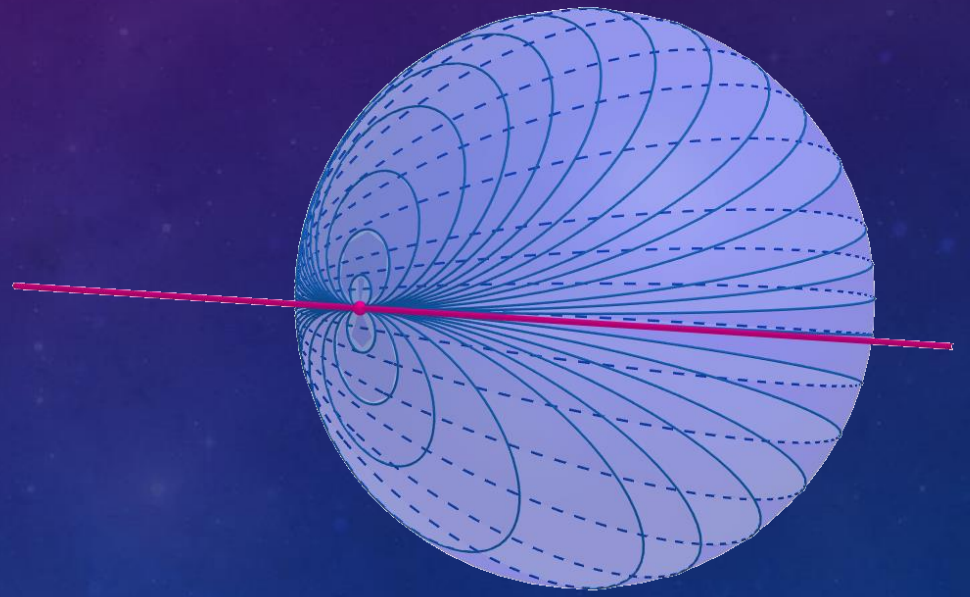

Tangent to $\mathbb{S}^2$ at point

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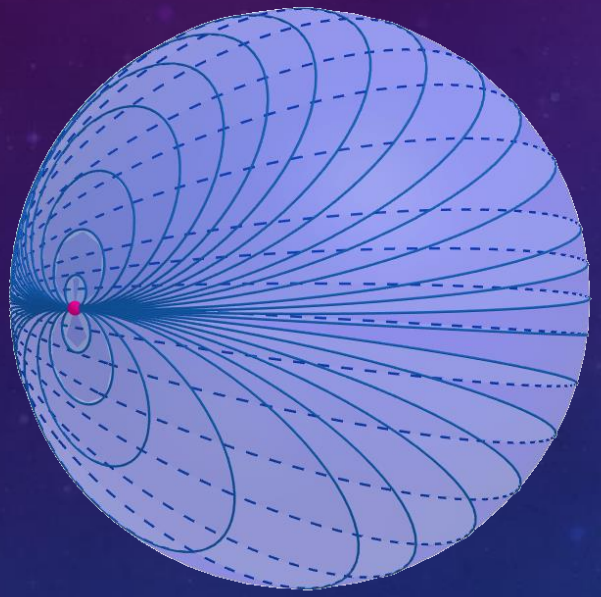

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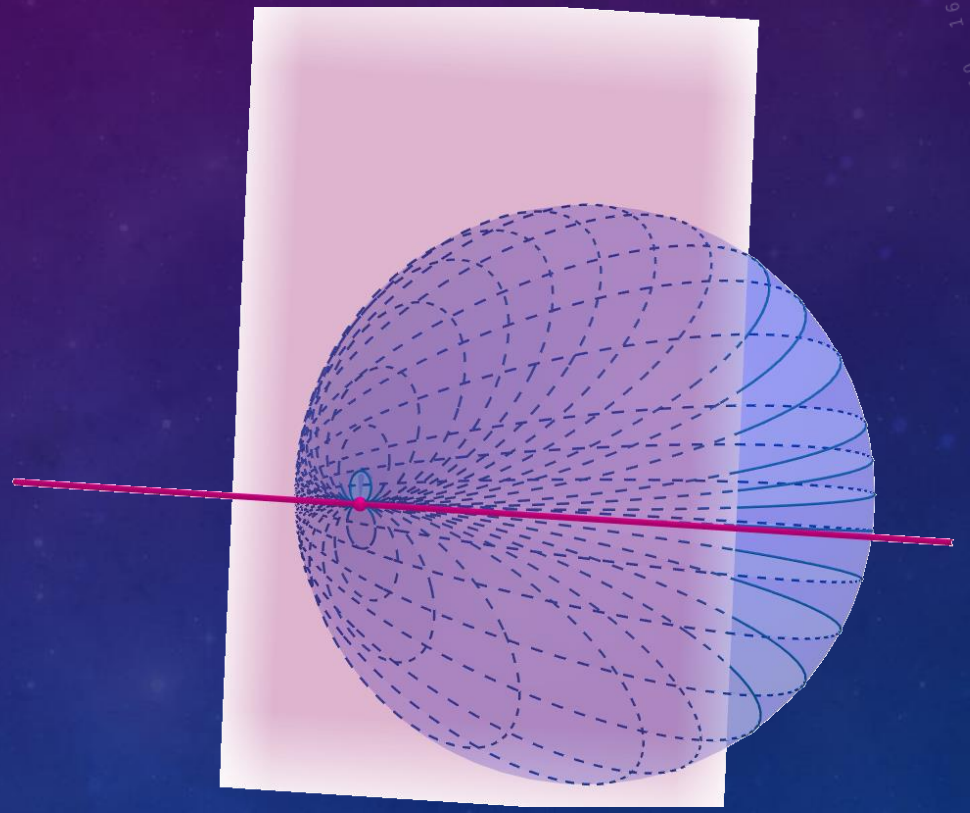

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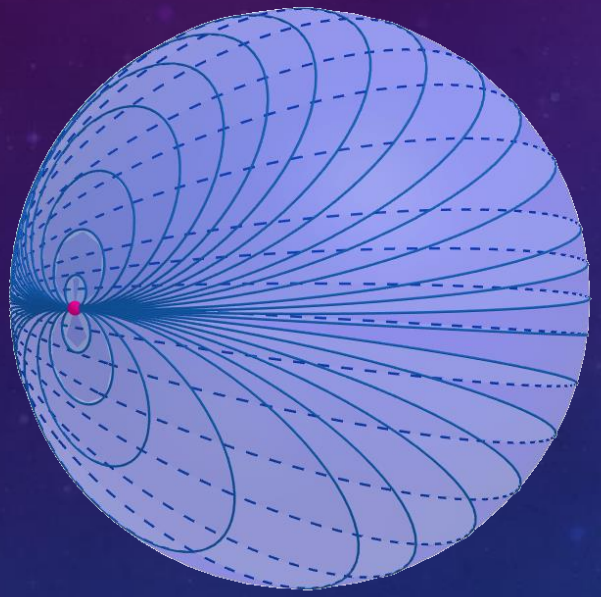

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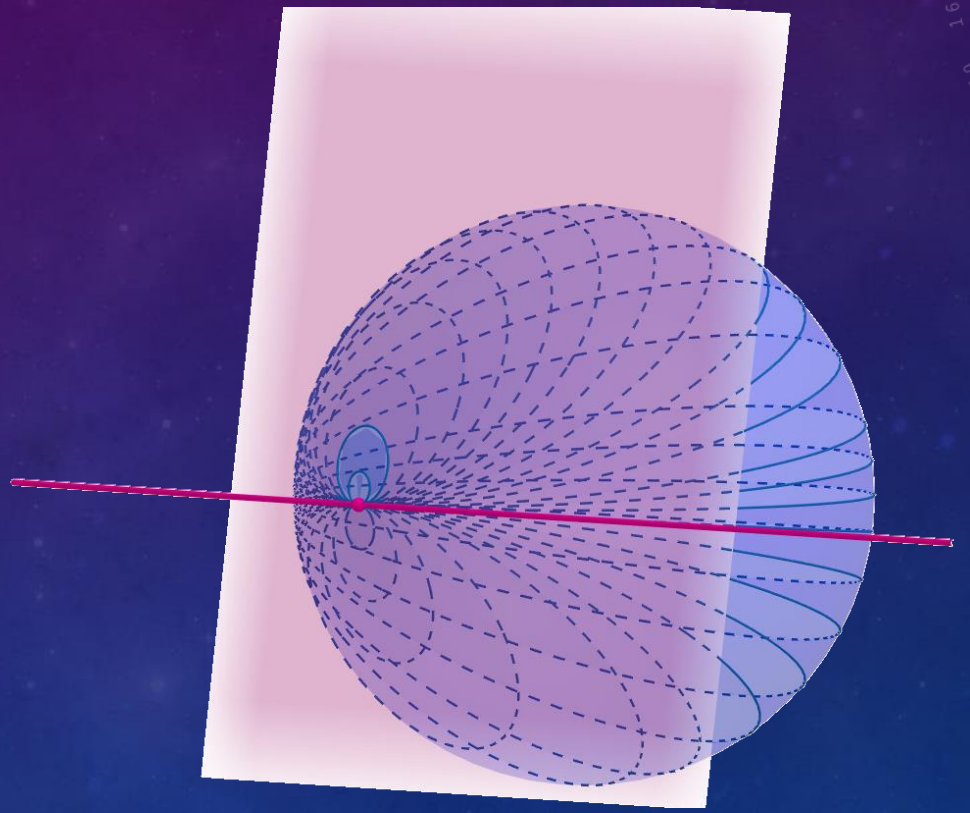

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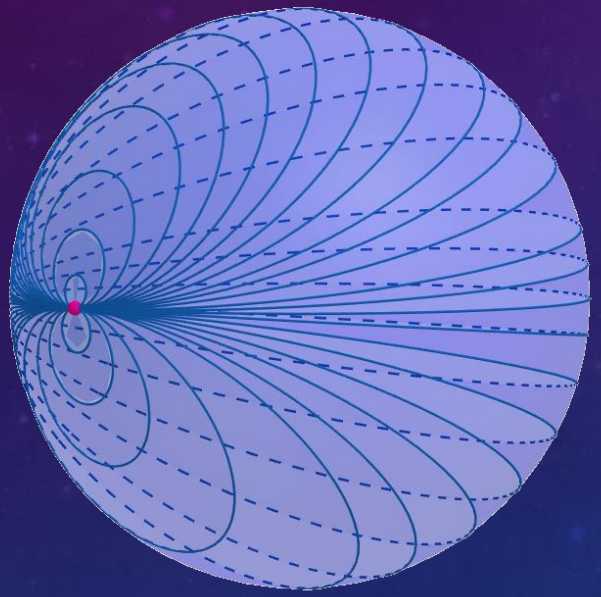

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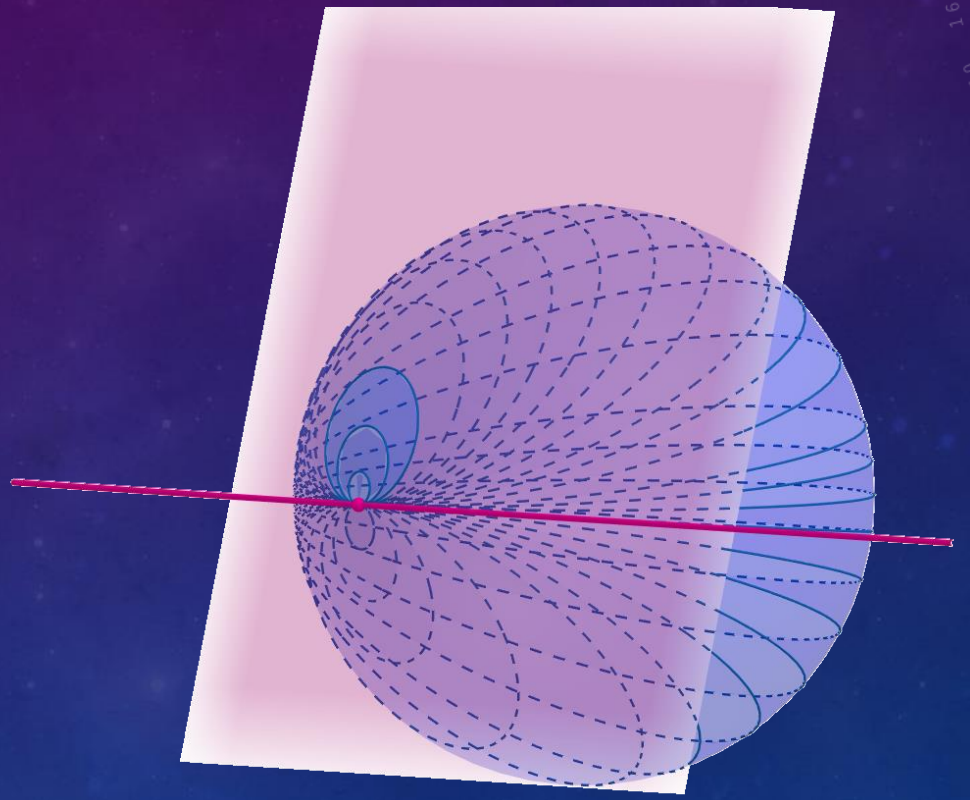

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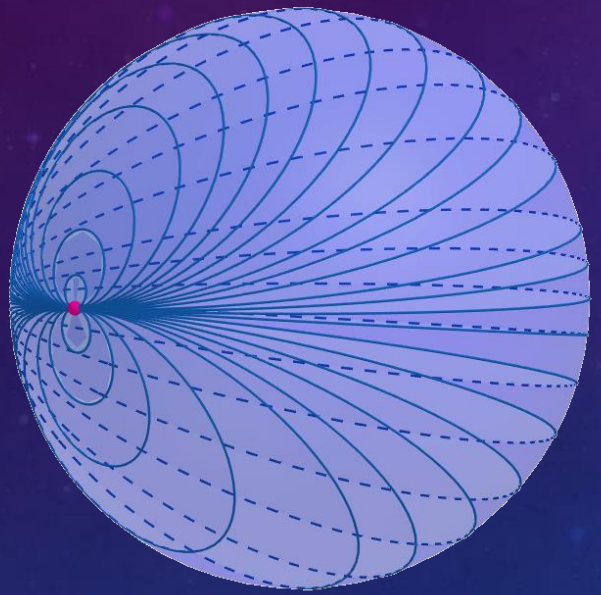

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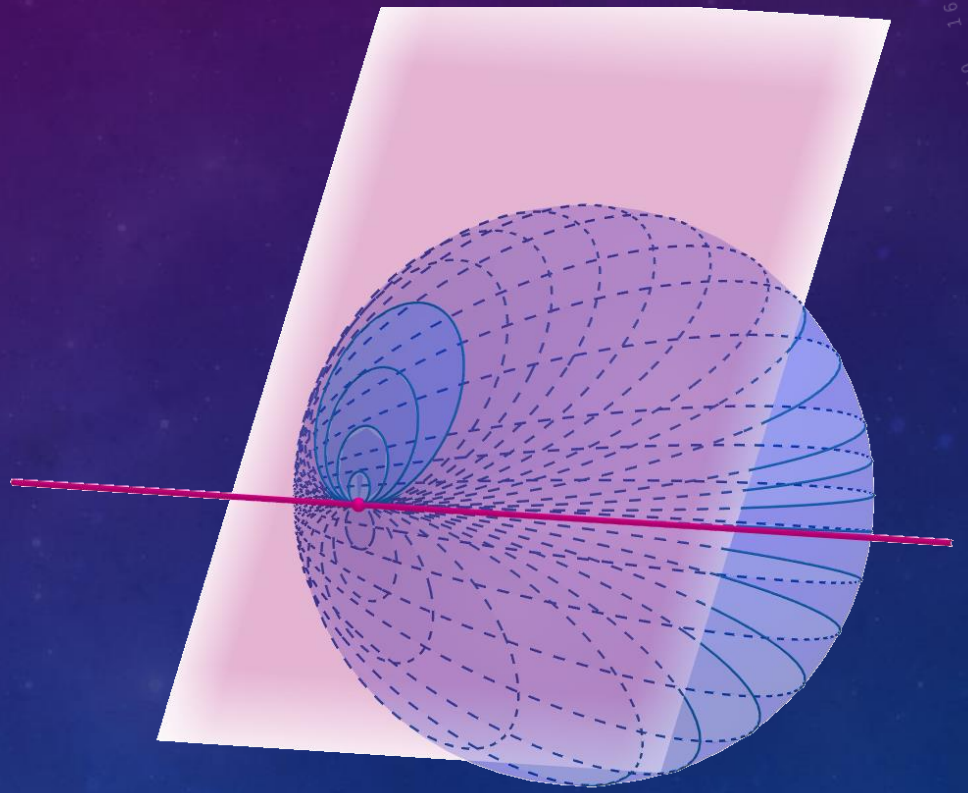

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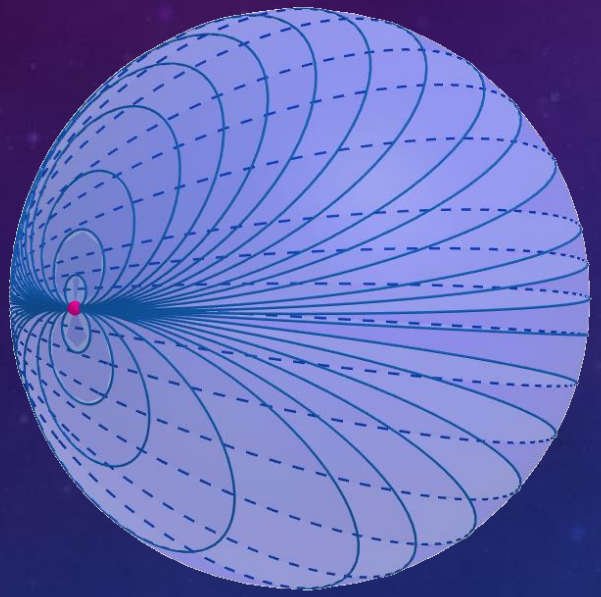

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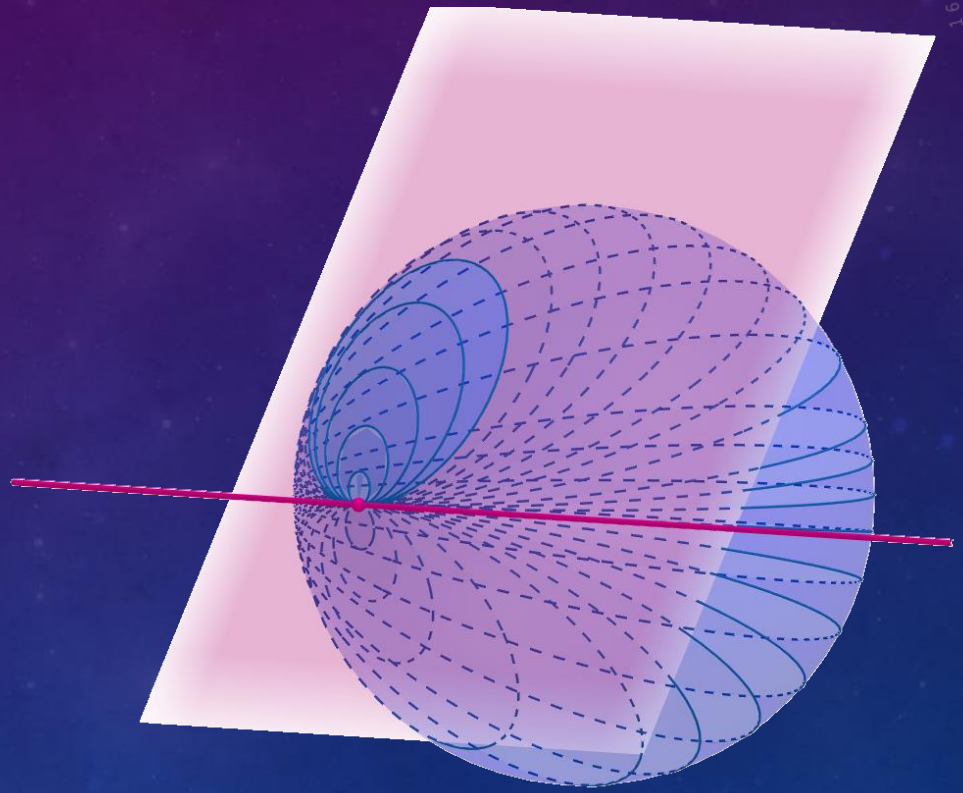

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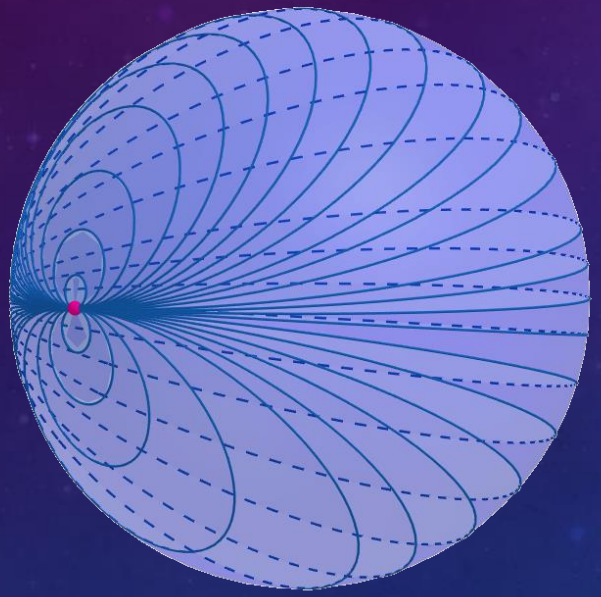

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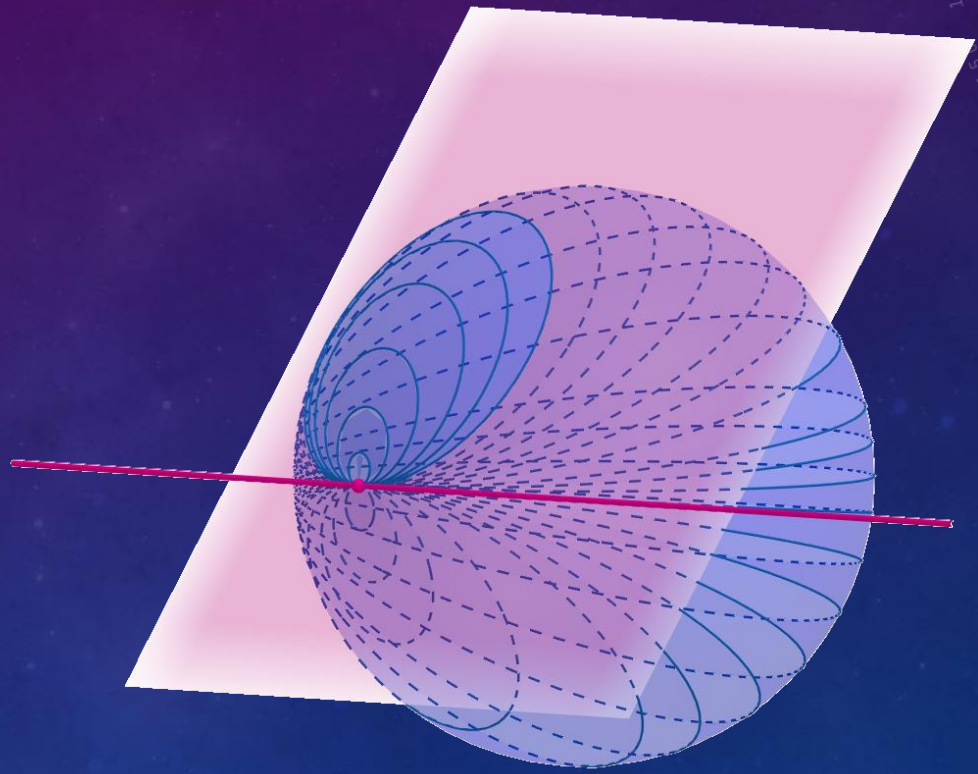

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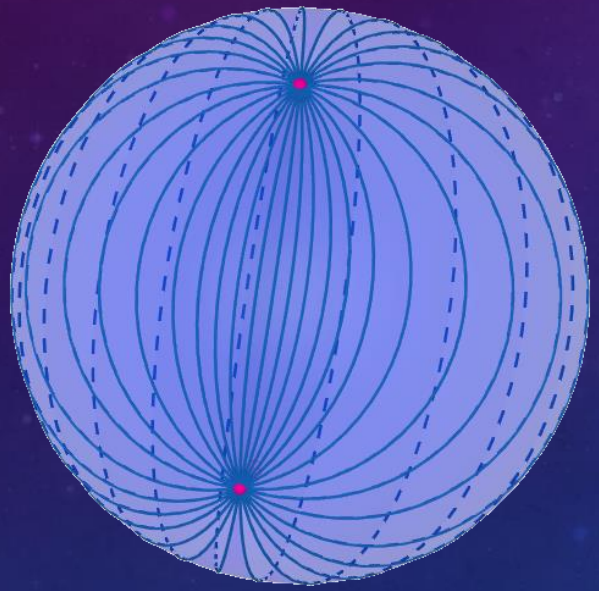

2. Intersect at two points

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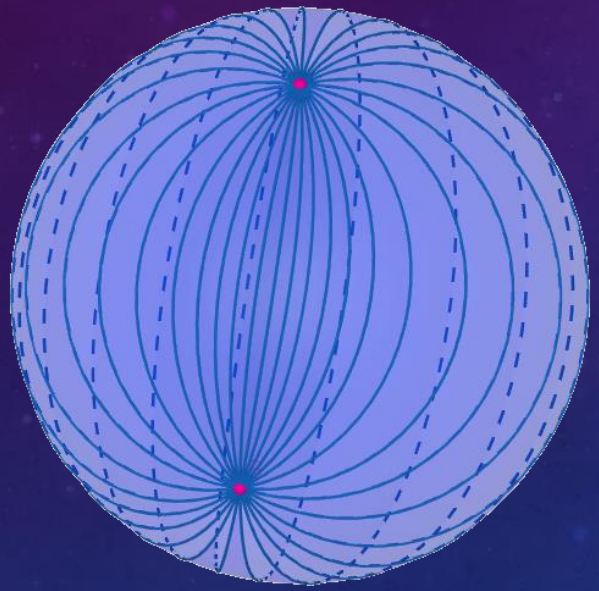

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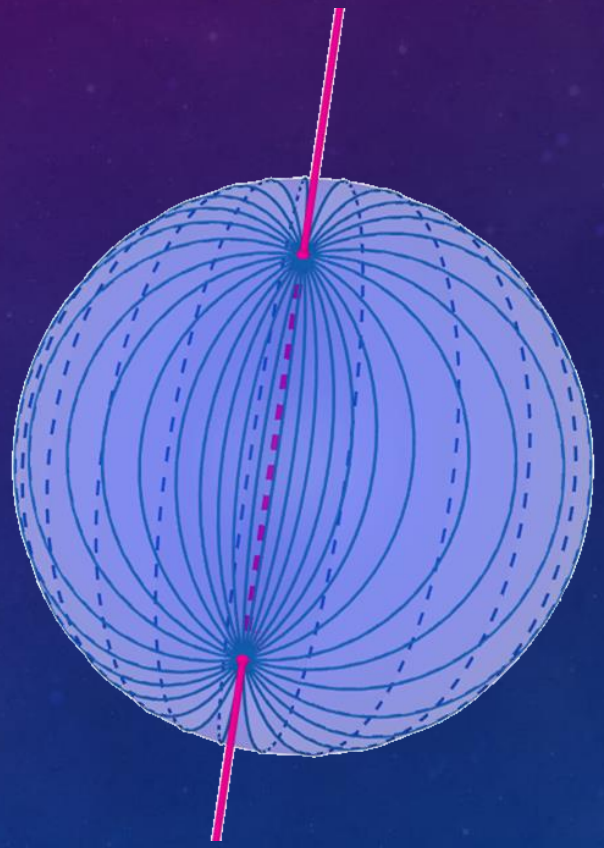

Intersects $\mathbb{S}^2$ at two points

C-LINES ↔ PROJECTIVE LINES

3. Don’t intersect

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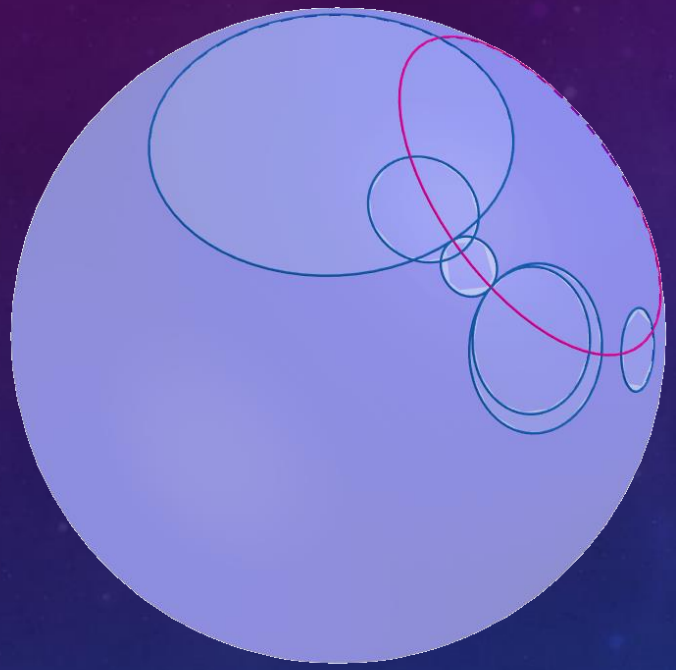
Doesn’t intersect $\mathbb{S}^2$.

C-PLANES



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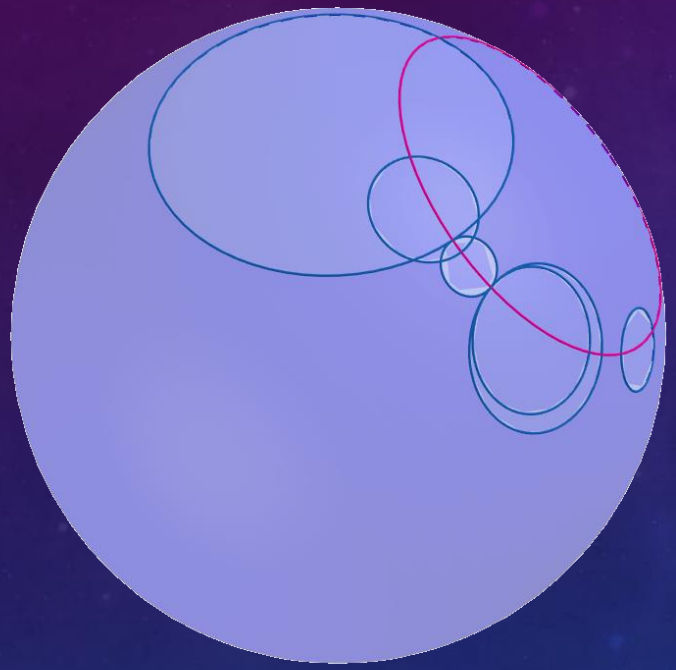
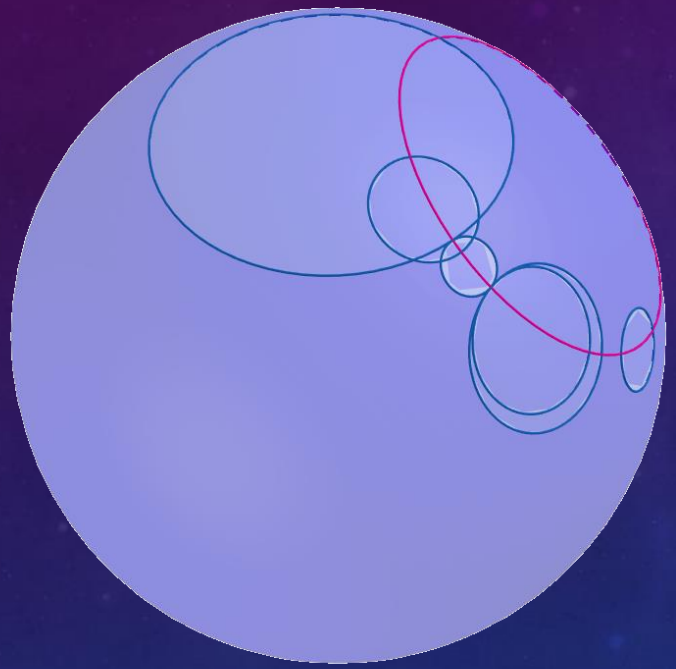


1. All circles orthogonal to generating circle

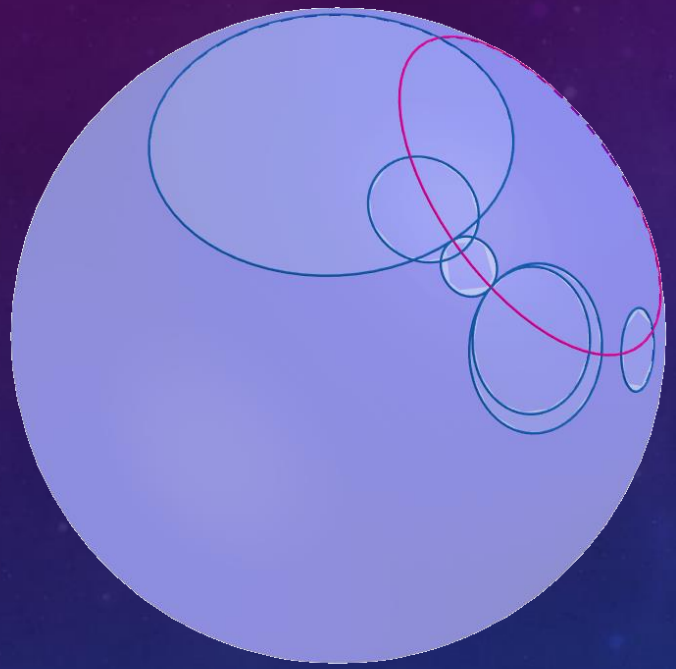
C-PLANES ↔ PROJECTIVE POINTS



1. All circles orthogonal to generating circle

Point outside sphere

C-PLANES ↔ PROJECTIVE POINTS



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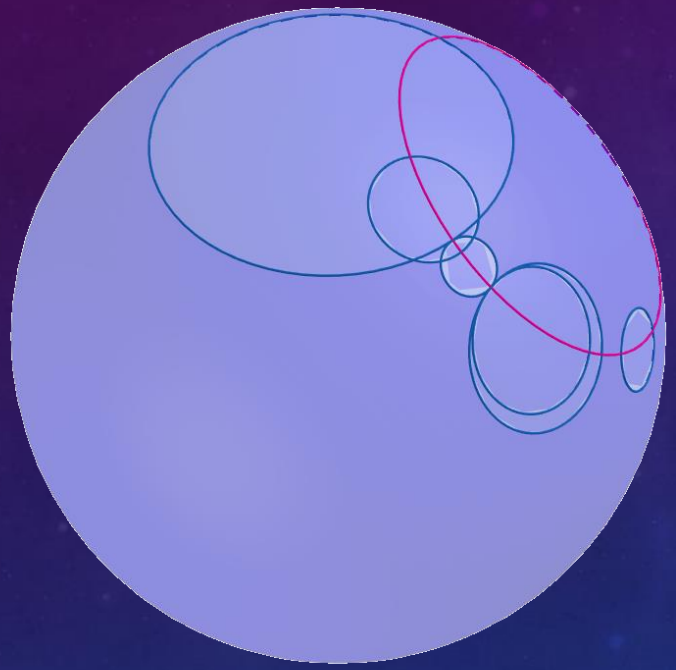

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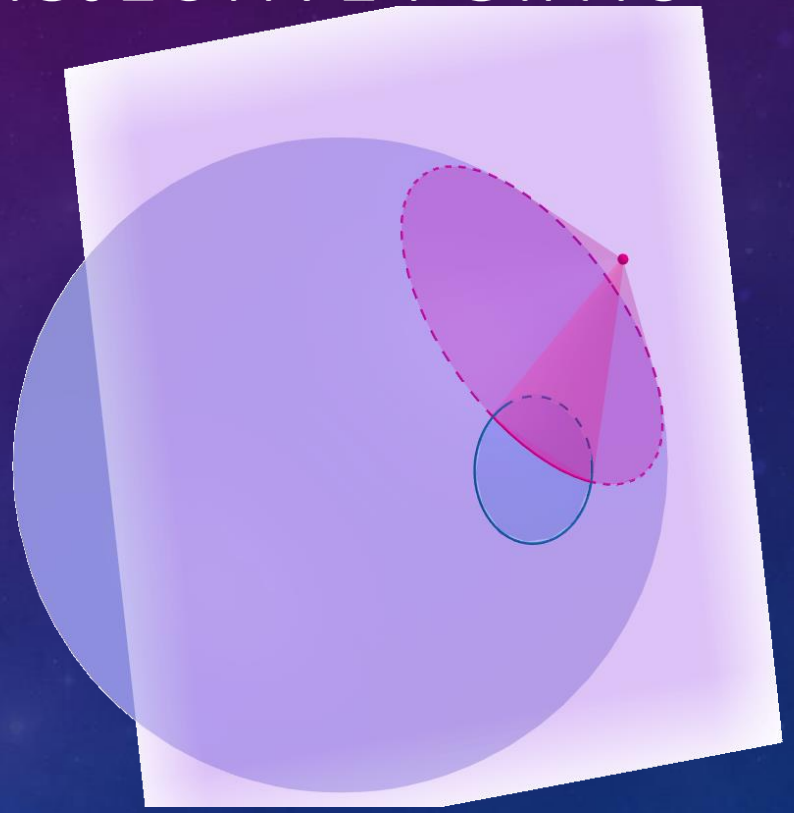

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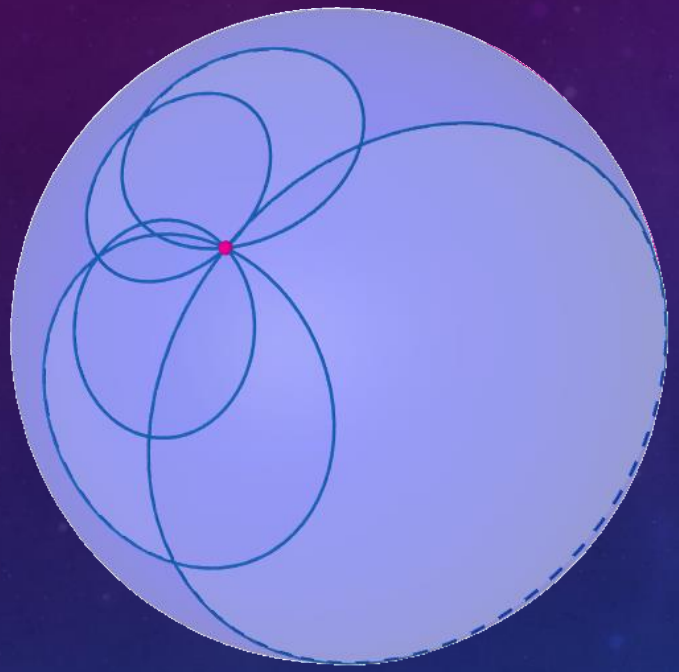
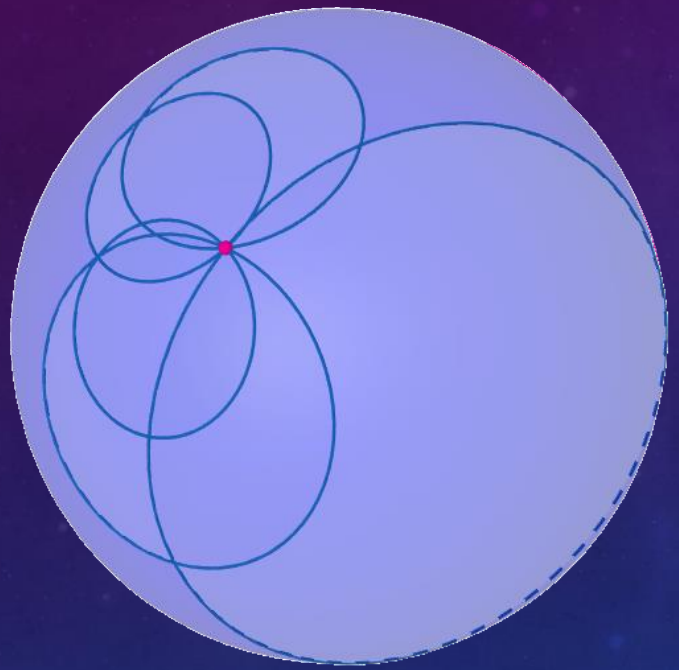


2. All circles through generating “circle”

C-PLANES ↔ PROJECTIVE POINTS



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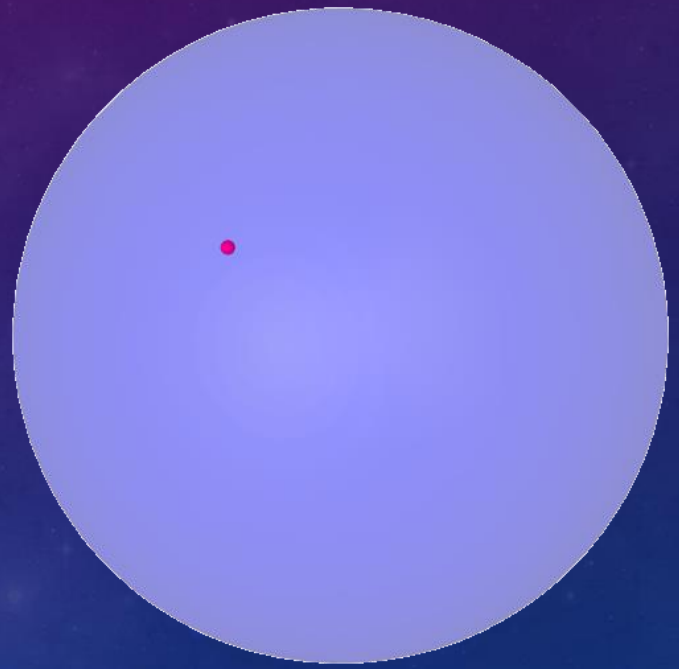

Point on sphere

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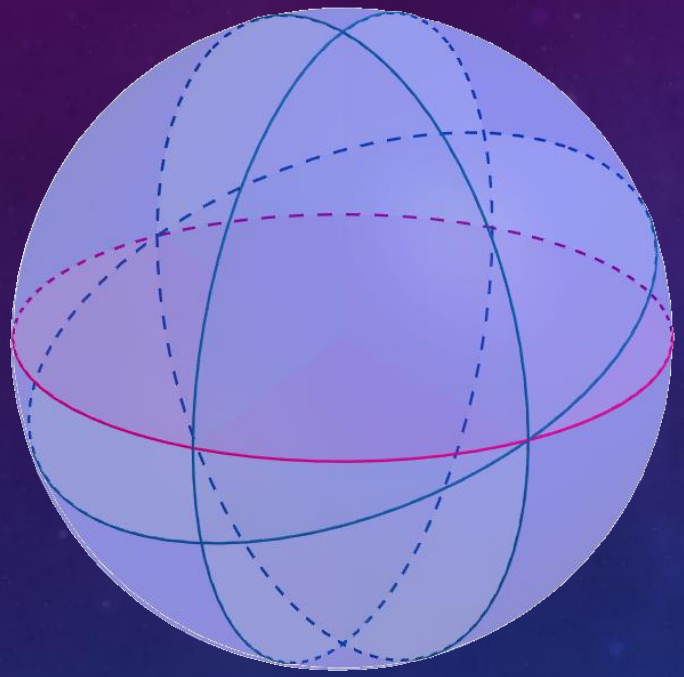
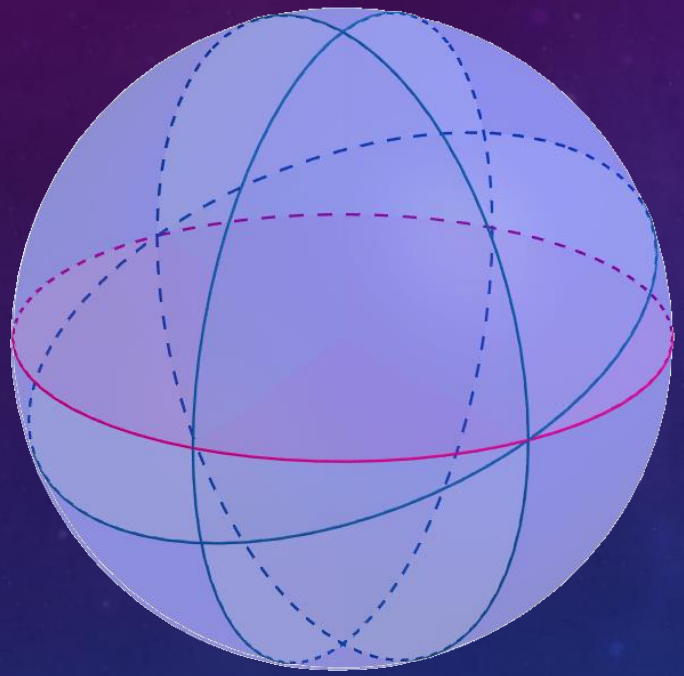


3. All circles through antipodal points on generating circle

C-PLANES ↔ PROJECTIVE POINTS



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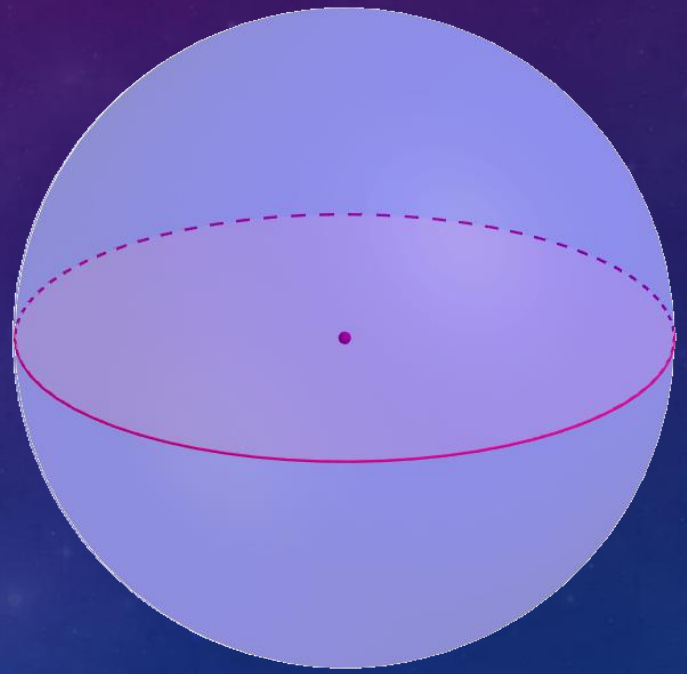
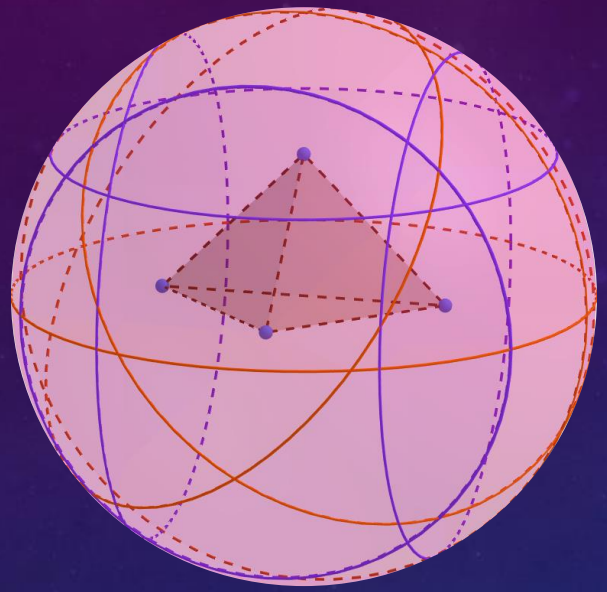


Point inside sphere

C-POLYHEDRA ↔ PROJECTIVE POLYHEDRA



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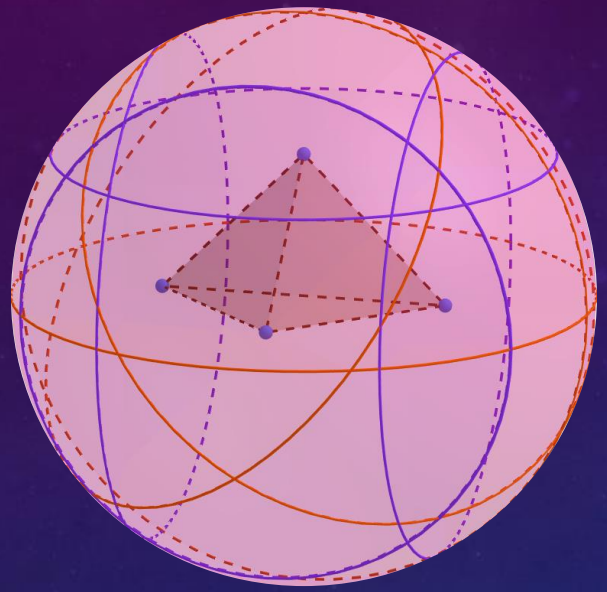
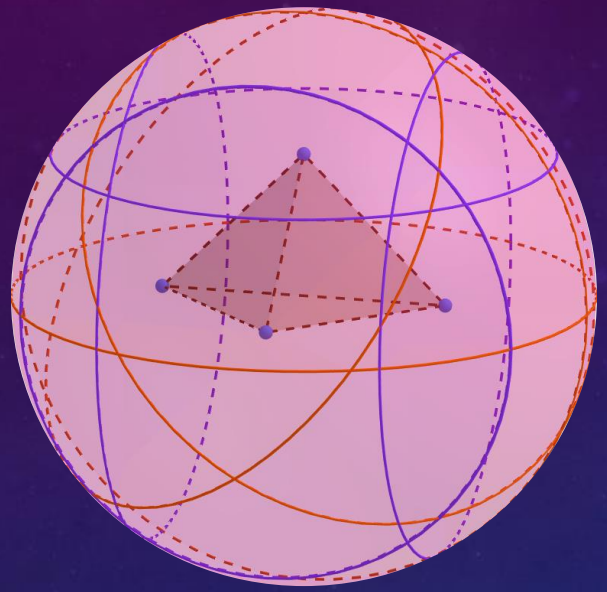


▲Hyperbolic polyhedra :

- Andre’ev (‘70)
- Rivin & Hodgson (‘93)

C-POLYHEDRA ↔ PROJECTIVE POLYHEDRA



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Ideal Polyhedra:
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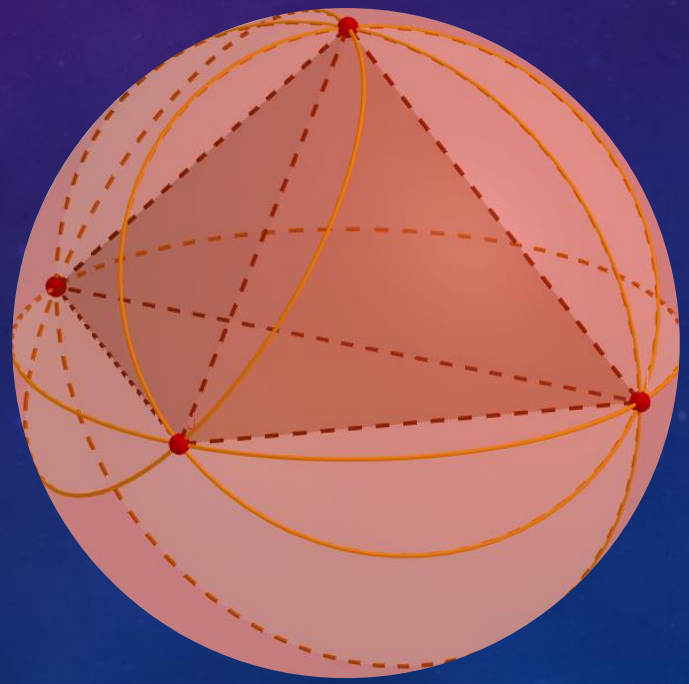

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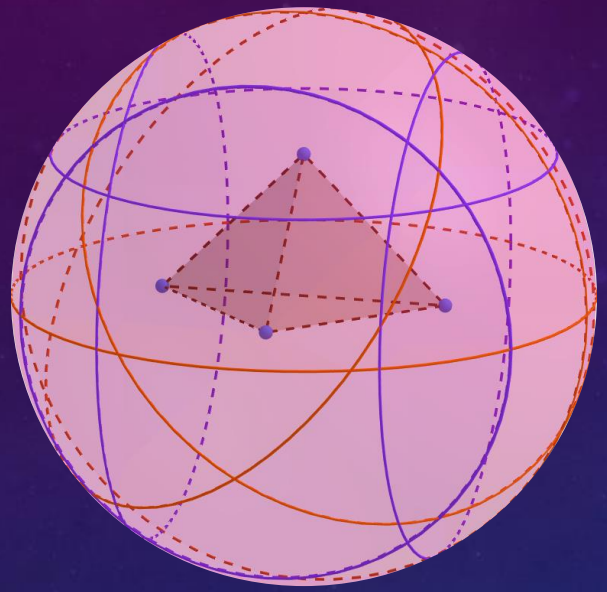

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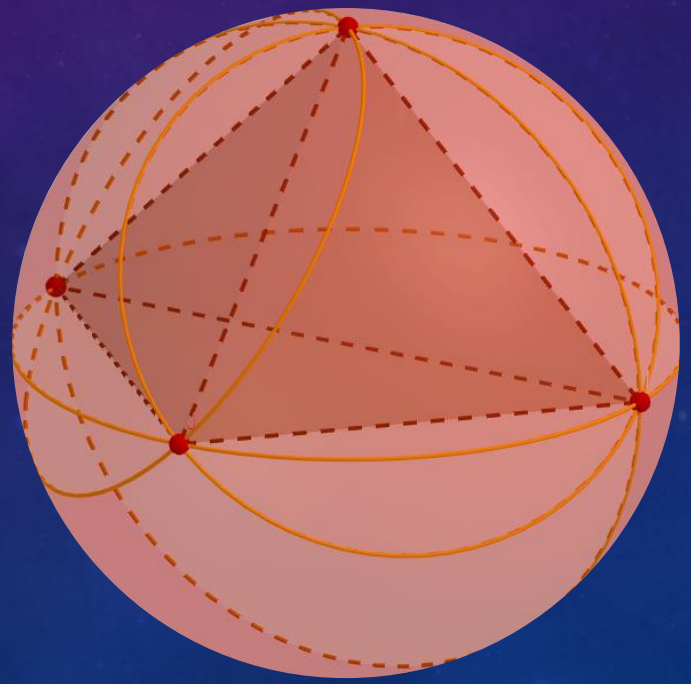

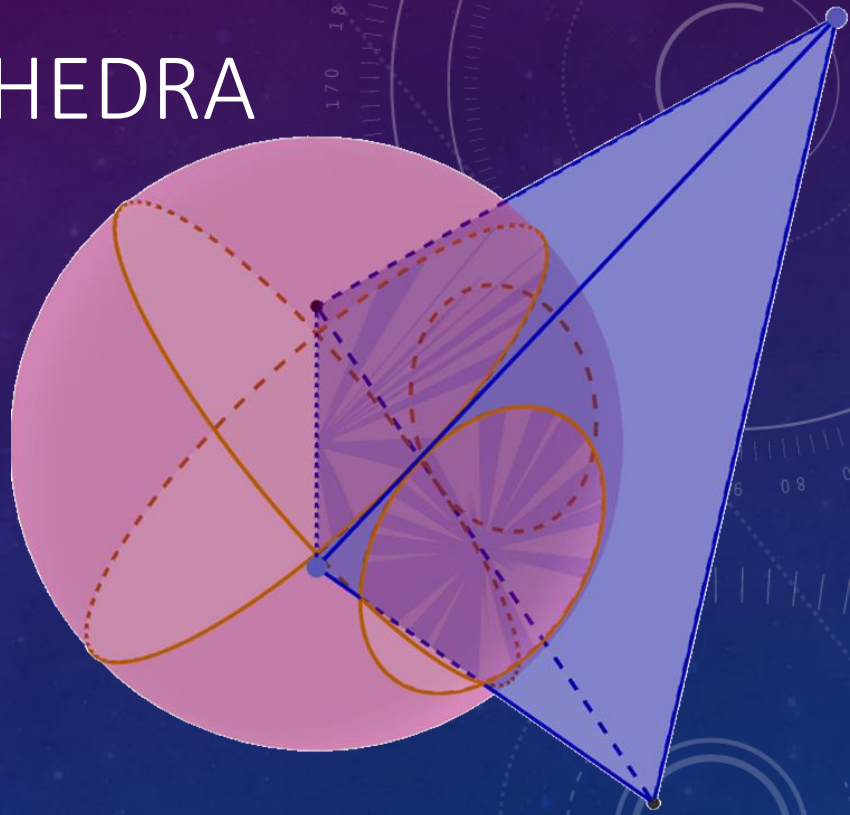

▲ Hyperideal polyhedra :

- Bao & Bonahon ('02)
- Bowers, Bowers, Pratt ('17)

“ALL GEOMETRY IS PROJECTIVE GEOMETRY.”
ARTHUR CAYLEY

“ALL GEOMETRY IS PROJECTIVE GEOMETRY.” ARTHUR CAYLEY

THANK YOU!

DUALITY

DUALITY

- c-points ↔ projective points

DUALITY

- c-points ↔ projective points
- c-lines ↔ projective lines

DUALITY

- c-points ↔ projective points
- c-lines ↔ projective lines
- c-planes → projective planes